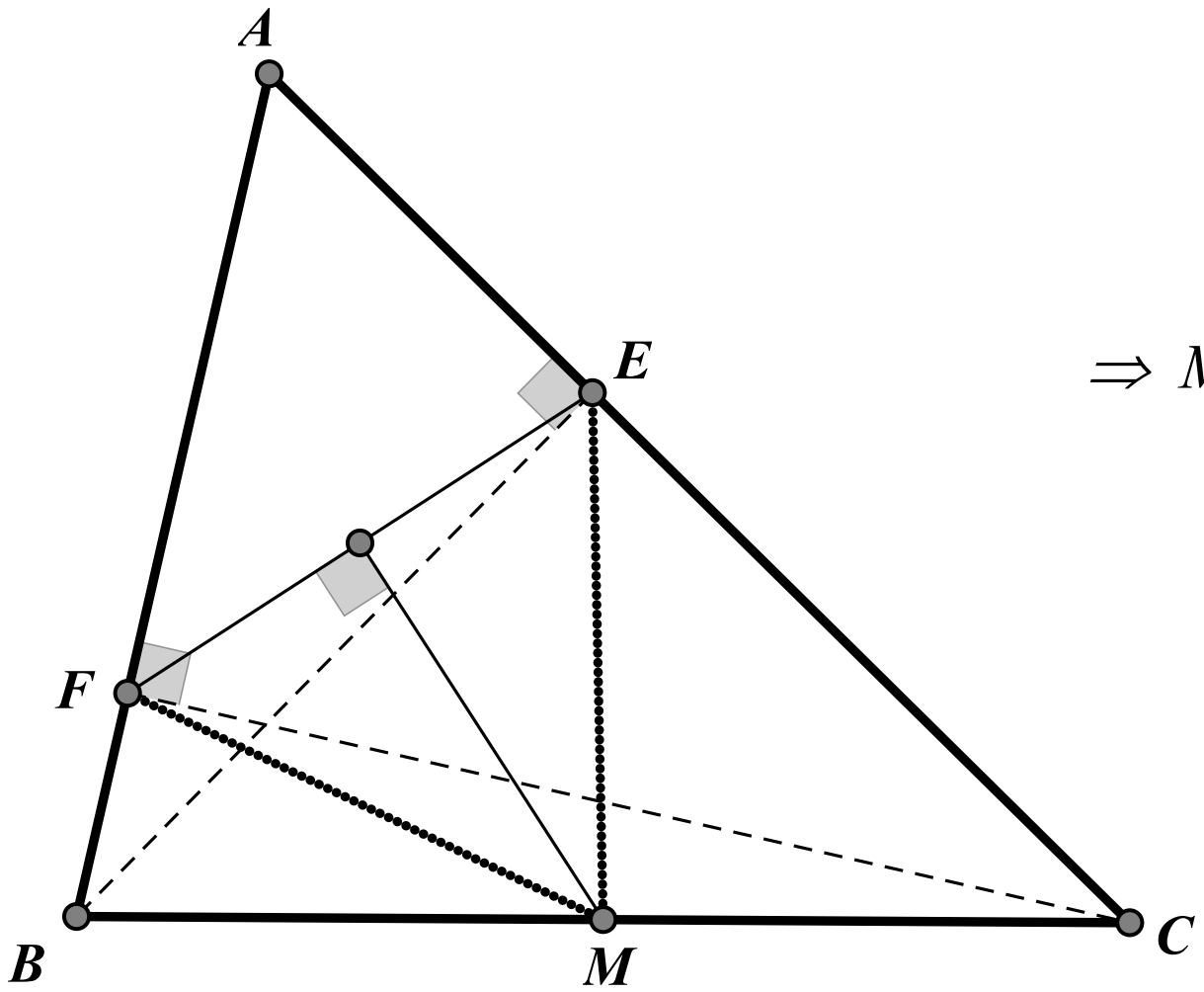
Triangles and Quadrilaterals

Some of my Favorite Problems, Part I

Robert Geretschläger

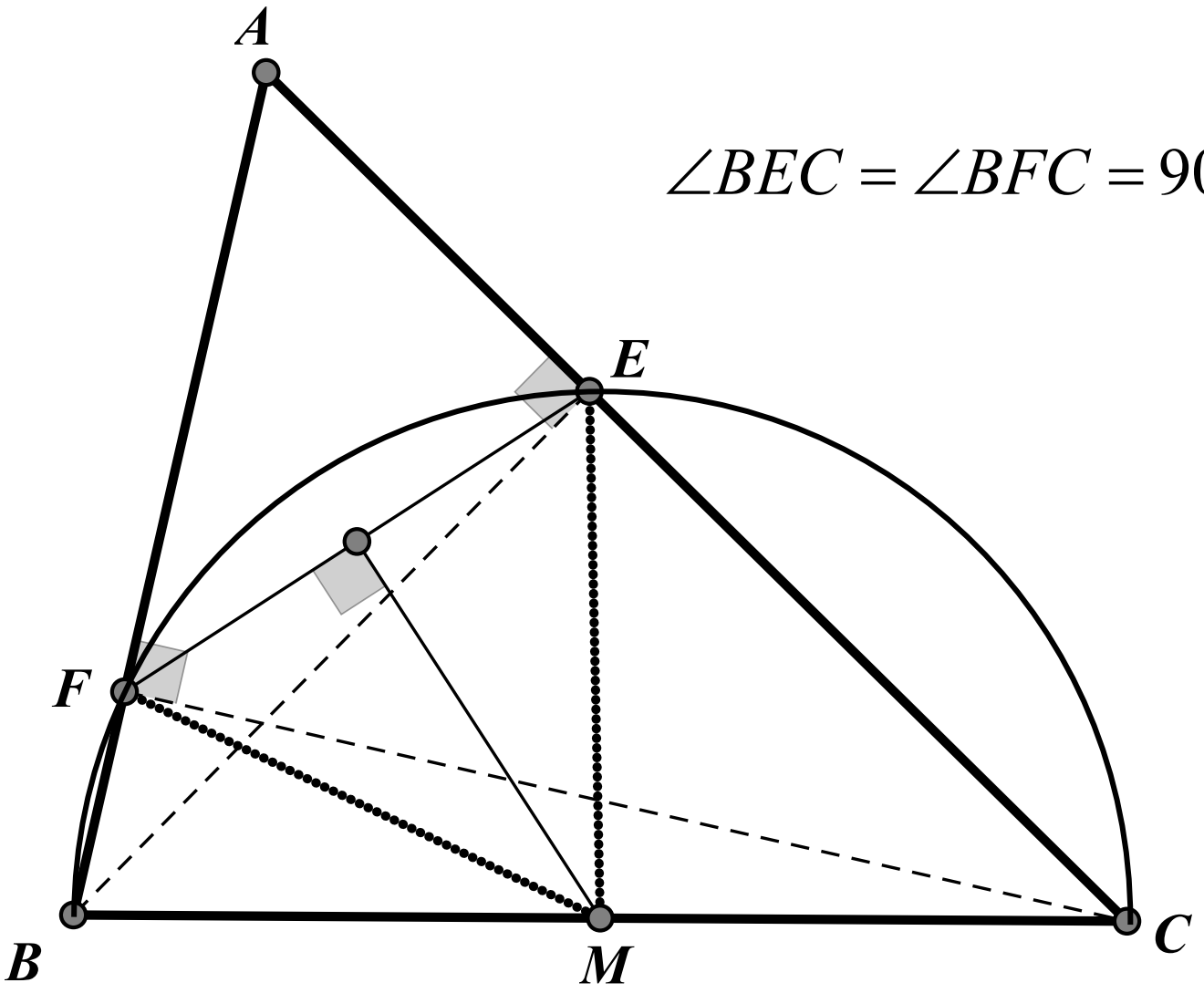
Graz, AUSTRIA



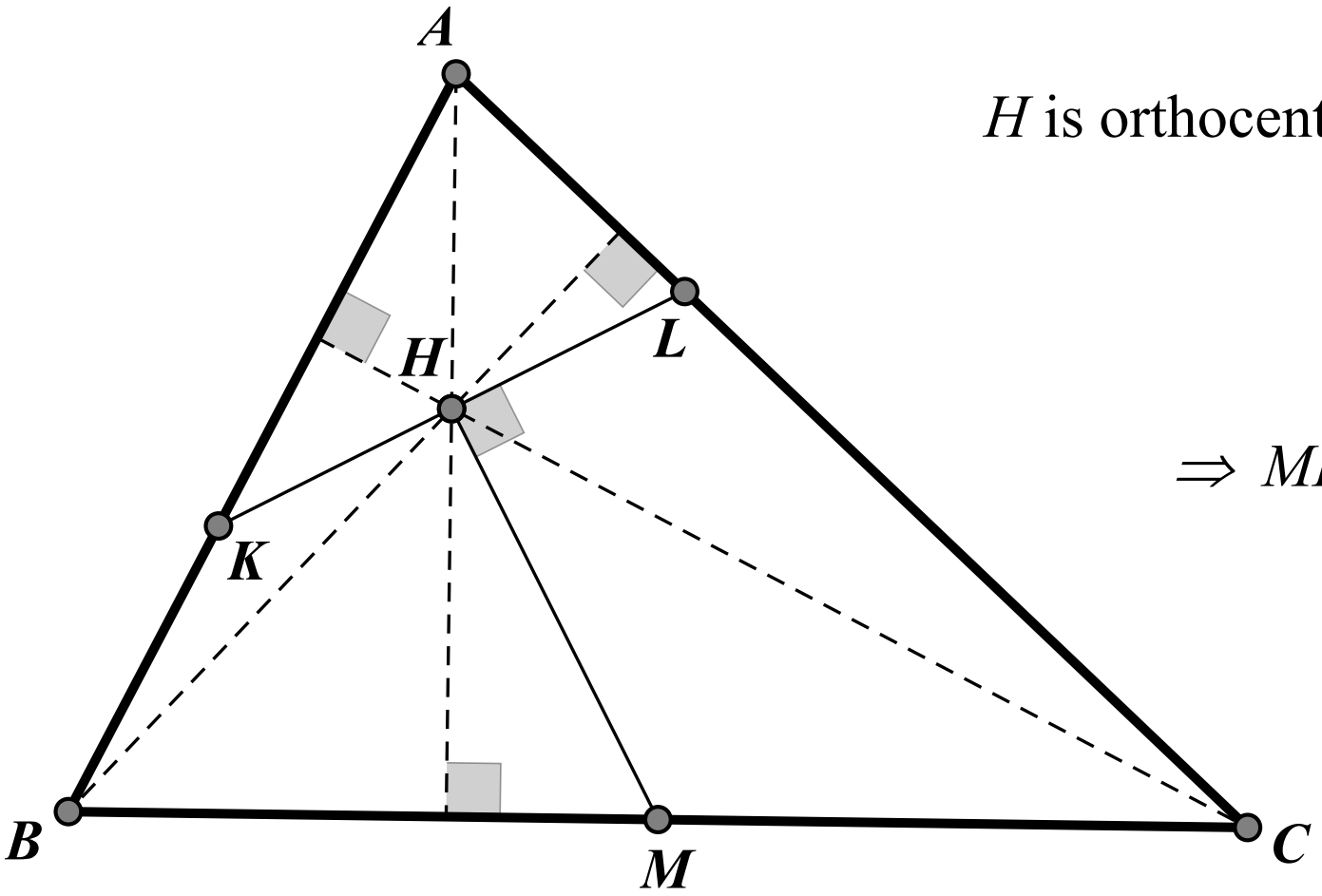
$$BE \perp CA,\ CF \perp AB$$

$$\Rightarrow MEF \text{ is isosceles with base } EF$$

Robert Geretschläger



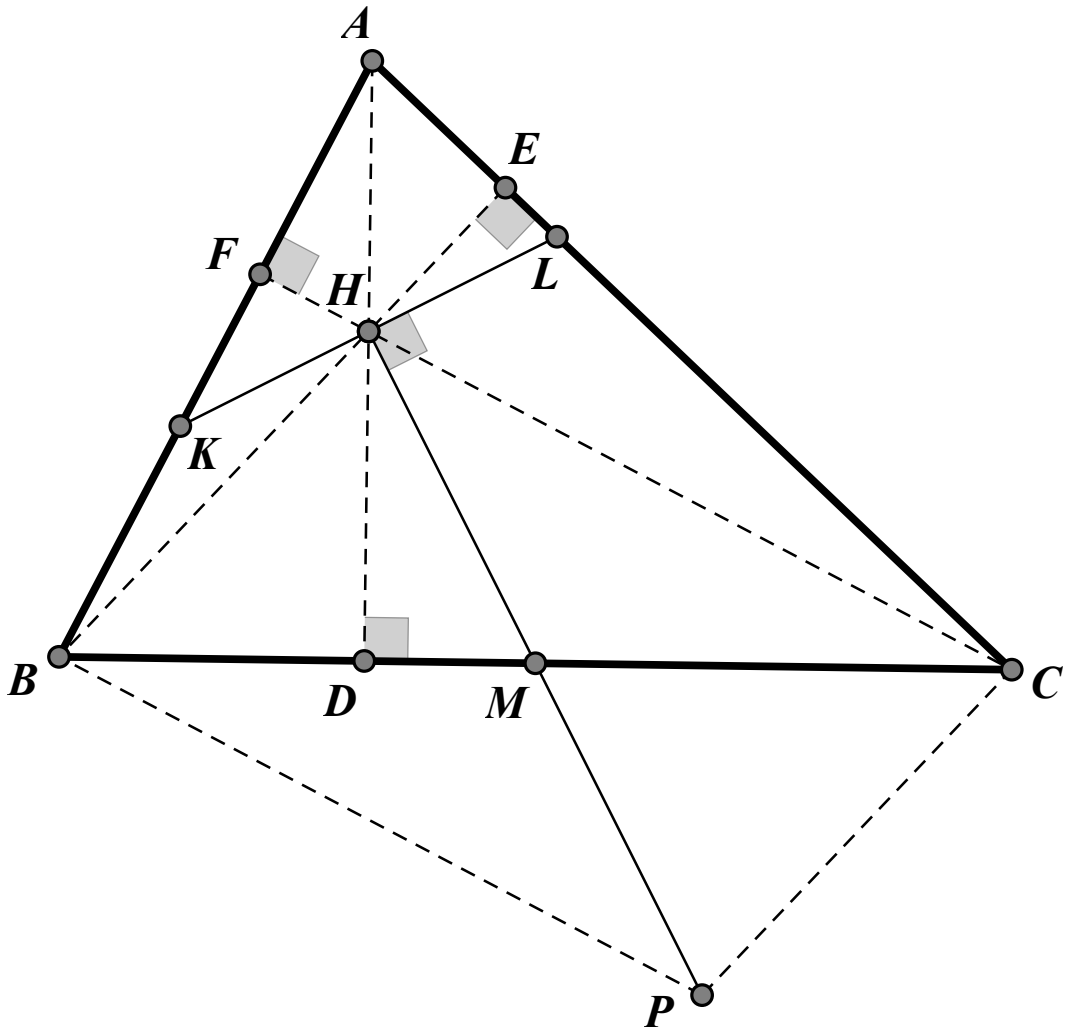
$$\angle BEC = \angle BFC = 90^\circ \Rightarrow MB = MF = ME = MC$$



H is orthocenter of ABC, M is midpoint of side BC

$$MH \perp KL$$

$\Rightarrow$ MLK is isosceles with base LK



$MP = MH \Rightarrow HBPC$ is a parallelogram

$\angle PBH = 180° - \angle BHC = 180° - \angle EHF = \angle FAE$

$\angle LKA = \angle HKF = 90° - \angle FHK$

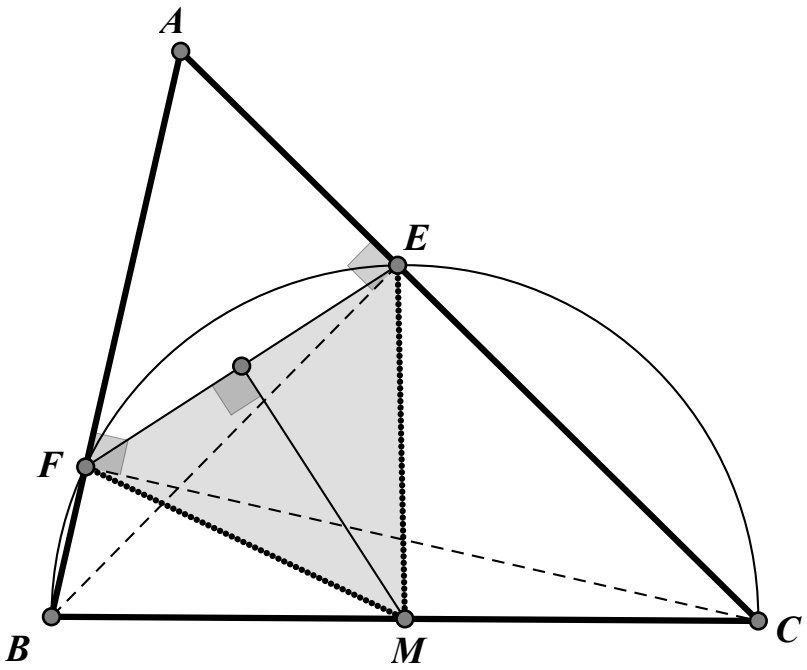
$= 90° - \angle CHL = \angle PHC = \angle HPB$

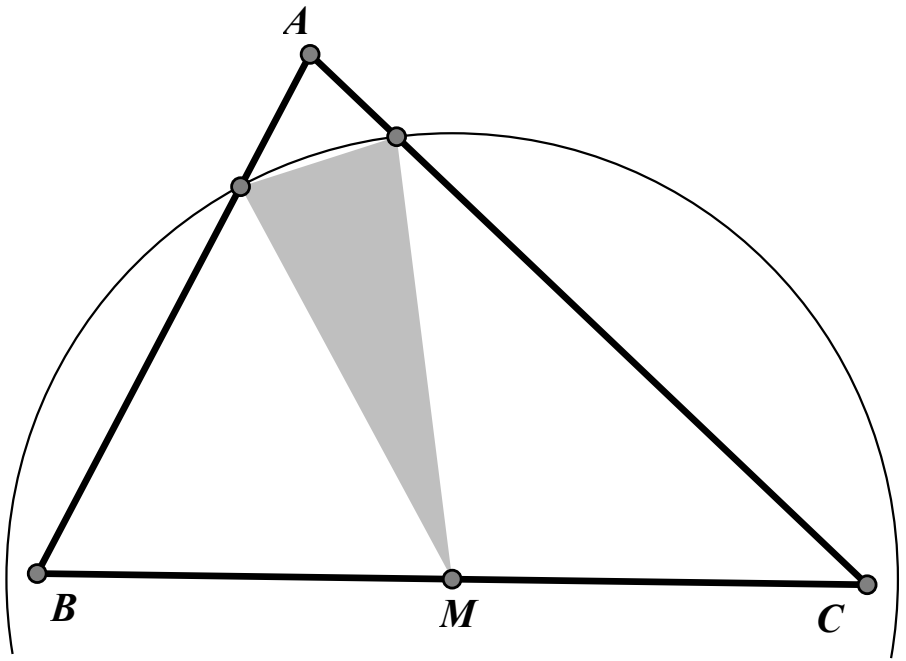
$\Rightarrow \Delta AKL \sim \Delta BPH$

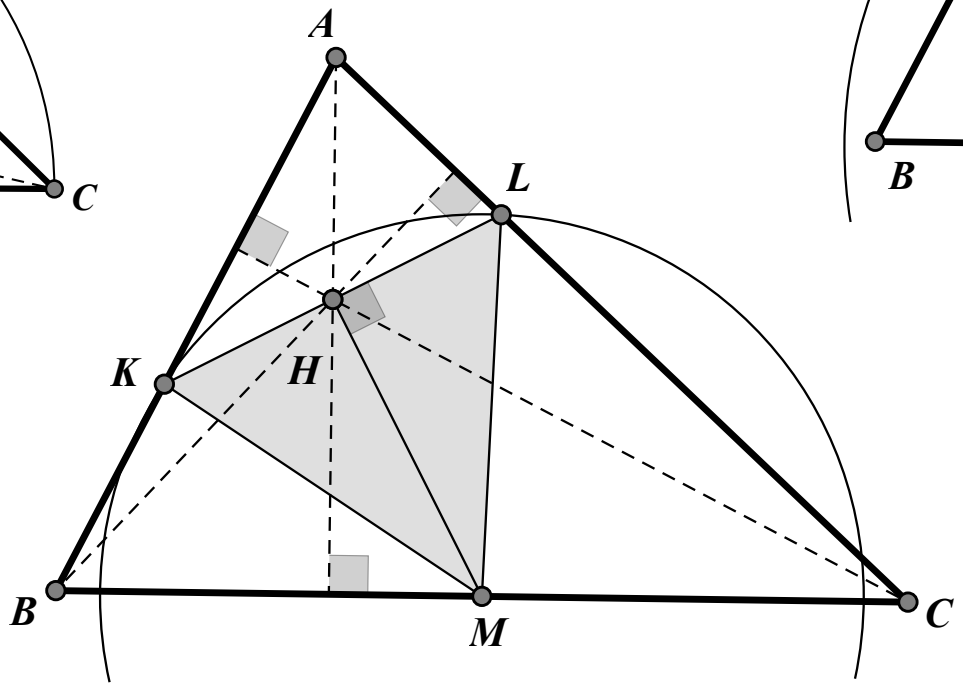
$\angle MBH = \angle CBE = 90° - \angle ACB = \angle DAC = \angle HAL$

$\Rightarrow KH : LH = PM : HM = 1 \Rightarrow KH = LH$

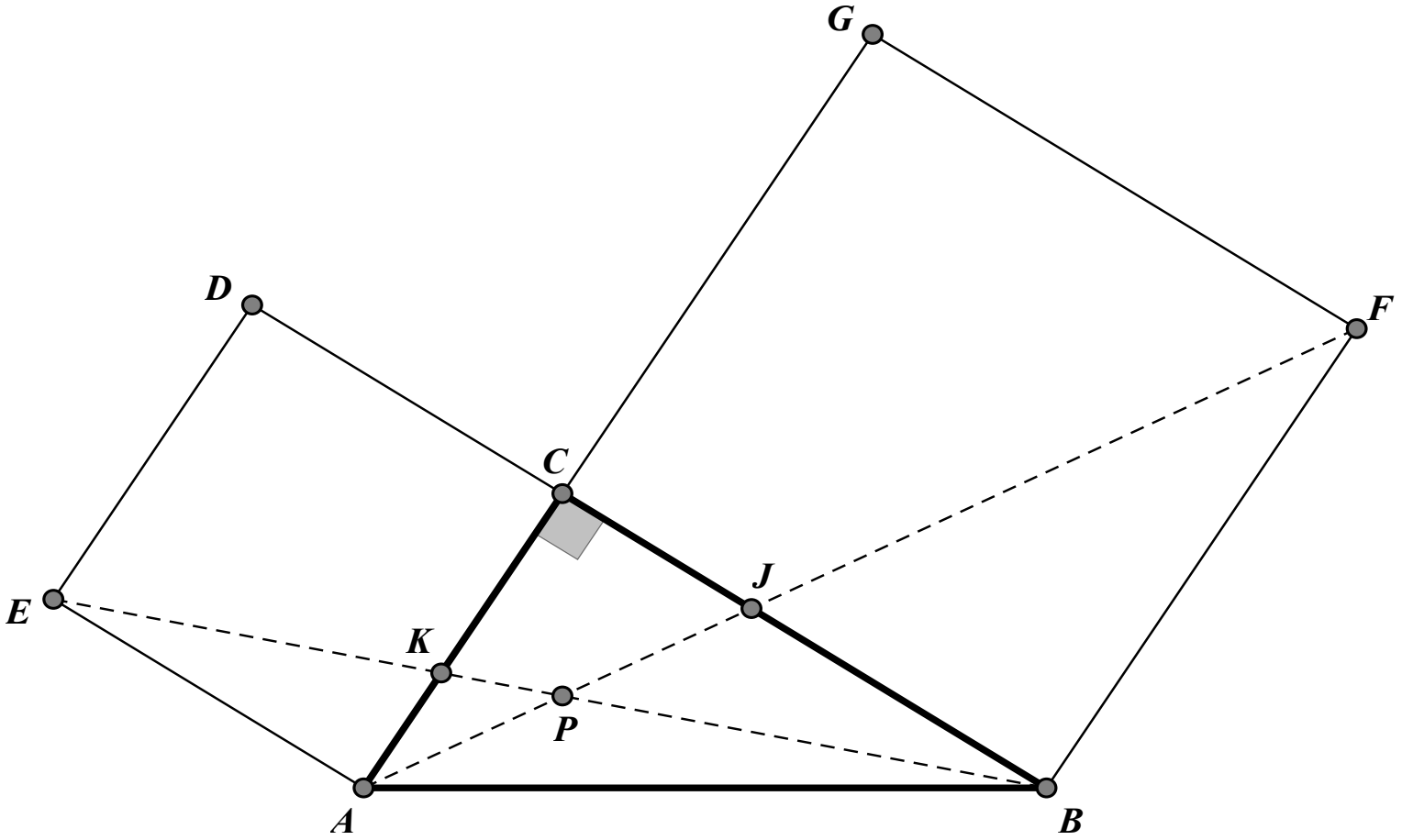
Robert Geretschläger





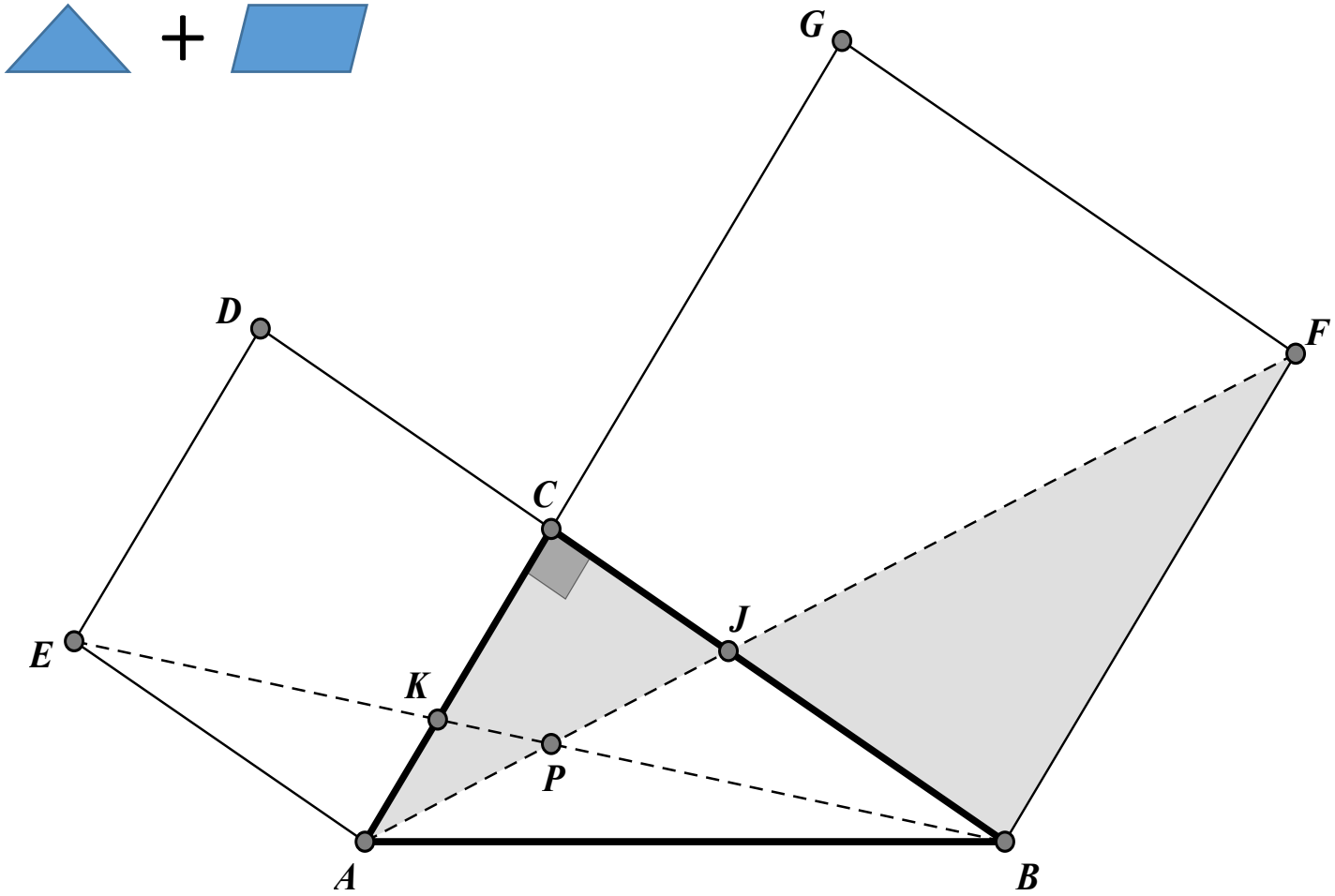


Robert Geretschläger



$$CK = CJ$$

Robert Geretschläger



$$\frac{CJ}{BJ} = \frac{AC}{BF} = \frac{b}{a}$$

$$BJ = BC - CJ = a - CJ$$

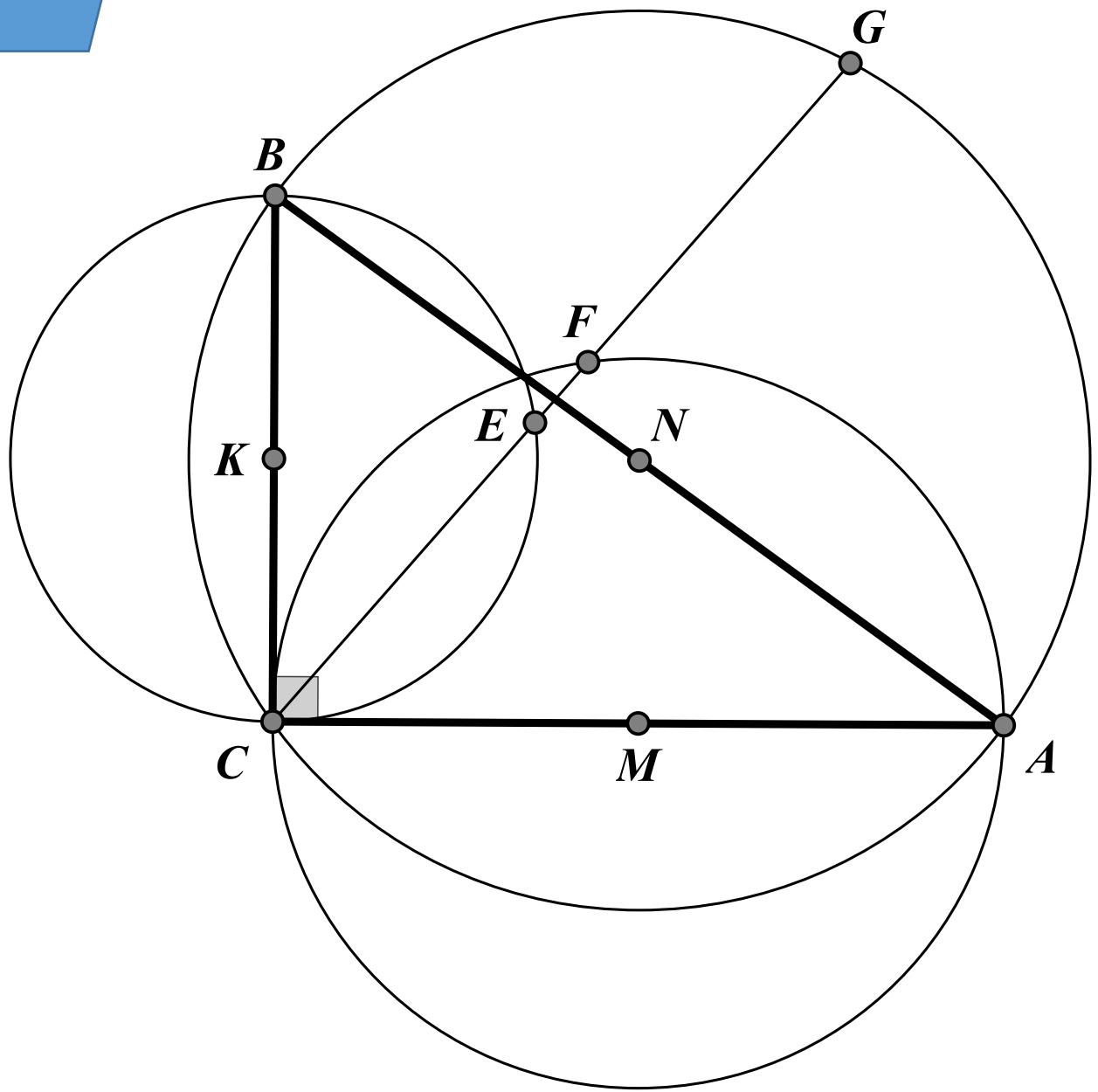
$$\Rightarrow \frac{CJ}{a - CJ} = \frac{b}{a}$$

$$\Leftrightarrow a \cdot CJ = ab - b \cdot CJ$$

$$\Leftrightarrow CJ = \frac{ab}{a + b}$$

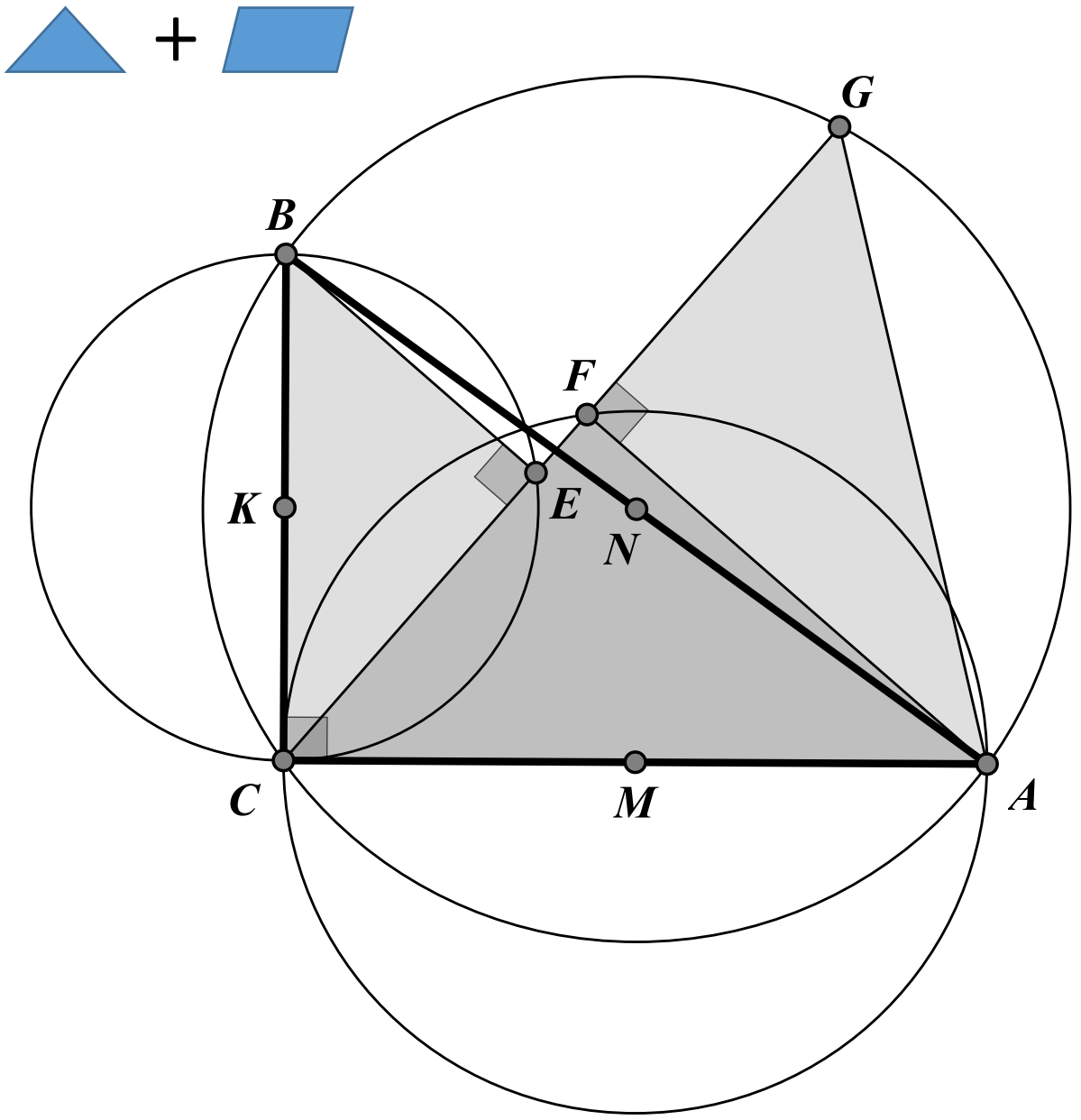
$$\frac{CK}{AK} = \frac{BC}{AE} = \frac{a}{b} \qquad \Rightarrow \frac{CK}{b - CK} = \frac{a}{b} \qquad \Leftrightarrow b \cdot CK = ab - a \cdot CK \qquad \Leftrightarrow CK = \frac{ab}{a + b} \qquad \Rightarrow CK = CJ$$

Robert Geretschläger



$$CE = FG$$

Robert Geretschläger



$$\angle BEC = 90°,\ \angle CFA = 90°,\ \angle AFG = 90°$$

$$\angle CGA = \angle CBA \quad \Rightarrow \Delta FAG \sim \Delta CAB$$

$$\Rightarrow FA = FG \cdot \frac{CA}{CB}$$

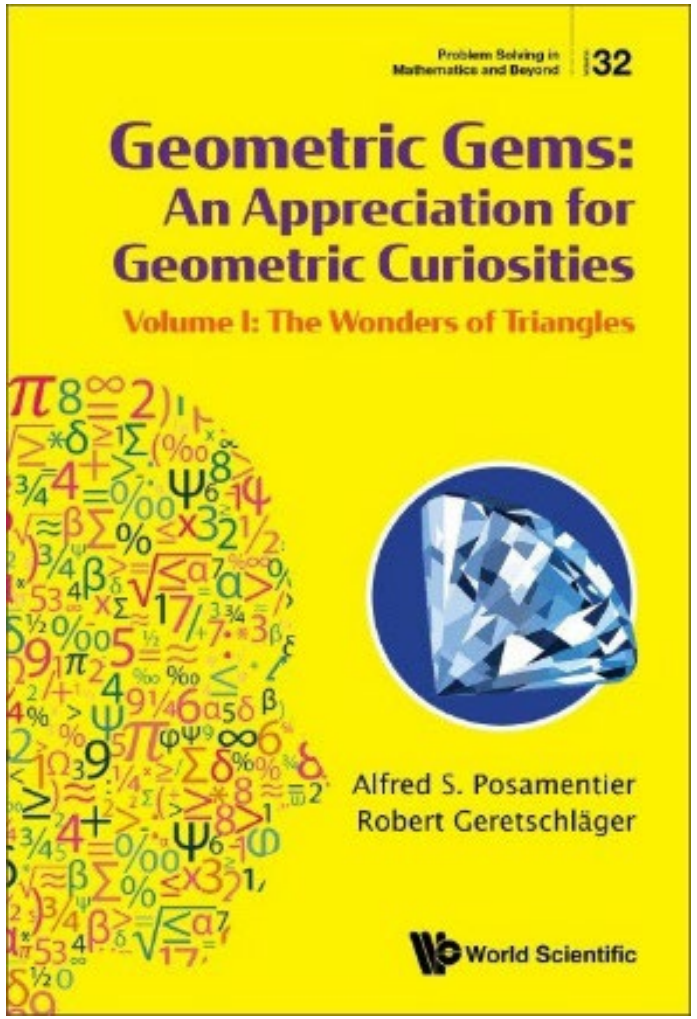
$$\angle ACF = \angle ACB - \angle ECB$$

$$= 90° - \angle ECB = \angle CBE$$

$$\Rightarrow \Delta EBC \sim \Delta FCA$$

$$\Rightarrow CE = FA \cdot \frac{CB}{CA}$$

$$\Rightarrow CE = FG \cdot \frac{CA}{CB} \cdot \frac{CB}{CA} = FG$$



www.rgeretschlaeger.com

robert@rgeretschlaeger.com

Danke für die Aufmerksamkeit