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Model Predictive Control for Autonomous Vehicles: A Simulation-Based Approach

Master's Thesis

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Graz, March 2025

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Datum	Unterschrift

List of Acronyms

NASA National Aeronautics and Space Administration

Kurzfassung

Kurze Beschreibung des Inhalts.

Abstract

Brief outlook on the content of this document.

Contents

Chapter 1

Introduction

Content of the Introduction. The Introduction 1 holds a figure 1.1, a figure 1.2 and cites something [1]. Afterwards it continues with chapter 2. The National Aeronautics and Space Administration (NASA) was founded in 1958. The NASA is known for its space missions.

Figure 1.1: Long caption text

(a) Caption figure A

(b) Caption figure B

Figure 1.2: Overall caption

1.1 Background

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1.1.1 Sub-section 1

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1.2 Problem Statement

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Chapter 2

Methodology

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2.1 A table

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Table 2.1: Constants of a system

Parameter	Value	Unit
m	0.19	kg
k	0.2	$\frac{\mathrm{kg}}{\mathrm{s}}$
c	4.8398	$\frac{\mathrm{N}}{\mathrm{m}}$

2.2 Some equations

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$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u, \qquad y = \boldsymbol{c}^T\boldsymbol{x} + du.$$

$$\begin{bmatrix} \boldsymbol{x}_{k+1} \\ u_k \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{A}_d & \boldsymbol{\Gamma}_1(\tau_a, \tau_b) \\ \boldsymbol{0} & 0 \end{bmatrix}}_{=:\boldsymbol{A}_\xi(\tau_a,\tau_b)} \underbrace{\begin{bmatrix} \boldsymbol{x}_k \\ u_{k-1} \end{bmatrix}}_{=:\boldsymbol{\xi}_k} + \underbrace{\begin{bmatrix} \boldsymbol{\Gamma}_0(\tau_a, \tau_b) \\ 1 \end{bmatrix}}_{=:\boldsymbol{b}_\xi(\tau_a,\tau_b)} u_k, \qquad y_k = \begin{bmatrix} \boldsymbol{c}^T & 0 \end{bmatrix} \boldsymbol{\xi}_k, \tag{2.1}$$

$$\begin{bmatrix} \boldsymbol{x}_{k+1} \\ u_k \\ u_{k-1} \\ \vdots \\ u_{k-\zeta+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{A}_d & \Gamma_1(\mathrm{Y}) & \dots & \Gamma_{\zeta-1}(\mathrm{Y}) & \Gamma_\zeta(\mathrm{Y}) \\ \boldsymbol{0} & 0 & \dots & 0 & 0 \\ \boldsymbol{0} & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{0} & 0 & \dots & 1 & 0 \end{bmatrix}}_{=:\boldsymbol{A}_\xi(\mathrm{Y})} \underbrace{\begin{bmatrix} \boldsymbol{x}_k \\ u_{k-1} \\ u_{k-2} \\ \vdots \\ u_{k-\zeta} \end{bmatrix}}_{=:\boldsymbol{\xi}_k}$$

$$+ \underbrace{\begin{bmatrix} \Gamma_0(\mathrm{Y}) \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{=:\boldsymbol{b}_\xi(\mathrm{Y})} u_k, \tag{2.2}$$

$$y_k = \begin{bmatrix} \boldsymbol{c}^T & 0 & 0 & \dots & 0 \end{bmatrix} \boldsymbol{\xi}_k.$$

2.3 Theorem

2.3.1 Theorem 1

Theorem 3.1: Pythagoras' theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the catheti.

$$a^2 + b^2 = c^2$$

In mathematics, the Pythagorean theorem, also known as Pythagoras' theorem, is a relation in Euclidean geometry among the three sides of a right triangle.

2.3.2 Theorem 2 with proof

Theorem 3.2

There exist two irrational numbers x, y such that x^y is rational.

Proof 3.1

If $x = y = \sqrt{2}$ is an example, then we are done; otherwise $\sqrt{2}^{\sqrt{2}}$ is irrational, in which case taking $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$ gives us:

$$\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}\sqrt{2}} = \sqrt{2}^{2} = 2.$$

List of Figures

List of Tables

Bibliography

[1] P. Peter, "Latex: A document preparation system," *Software: Practice and Experience*, vol. 23, no. 10, pp. 141–152, 1995.