Robot Vision:
Projective Geometry - Vanishing points

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SS 2025

Learning goals

- Understand the concept of vanishing points and vanishing lines
- Understand the calculation of vanishing points
- Understand the relation between vanishing points and camera orientation and calibration

Outline

- Vanishing points and lines
- Applications of vanishing points

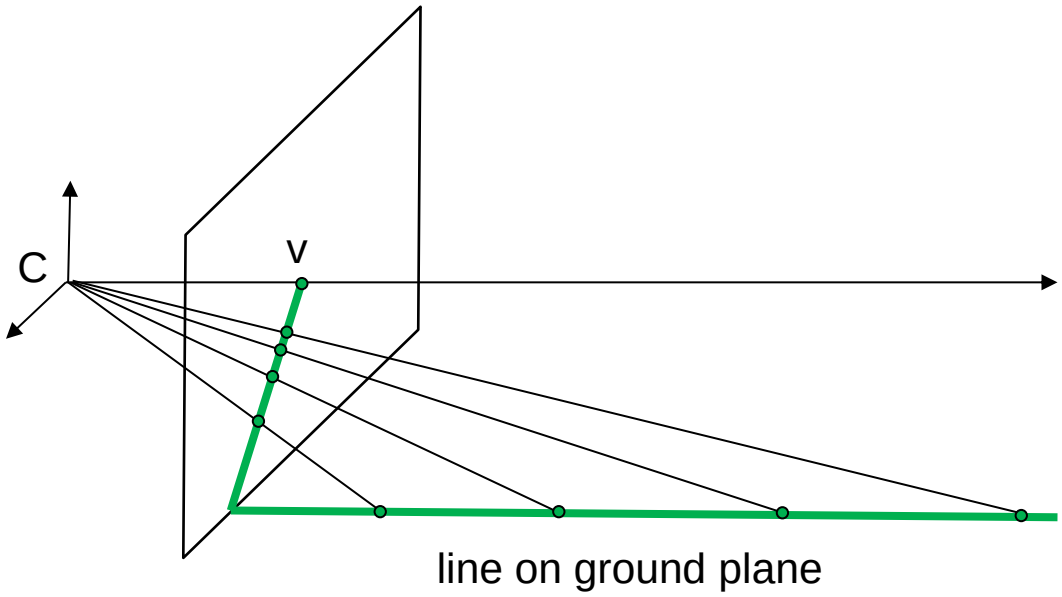
Vanishing points

Vanishing points

- Ideal points in the projective 3-space are located at infinity, and have homogeneous coordinates of the form (x, y, z, 0).
- These points are also called points at infinity.
- The image of an ideal point under a projective mapping is called a vanishing point.
- Recall that two parallel lines in the projective 3- space meet at an ideal point.
- Thus the images of two or more parallel world lines converge at a vanishing point.

[Source: Flickr]

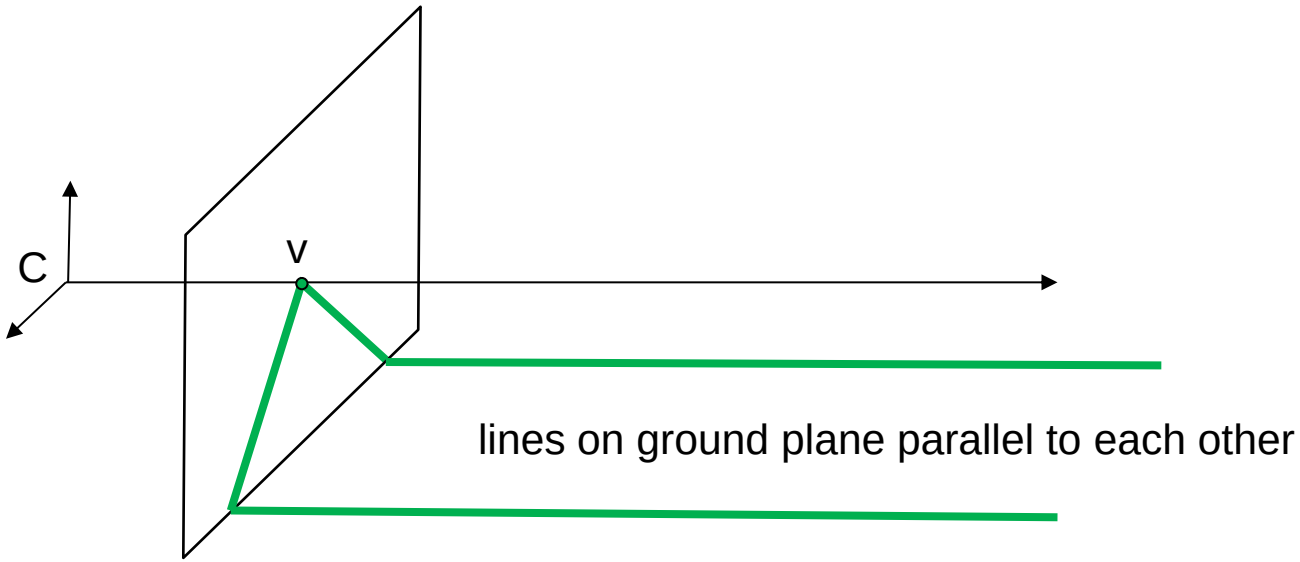
Vanishing points



The vanishing point v is the projection of a point at infinity.

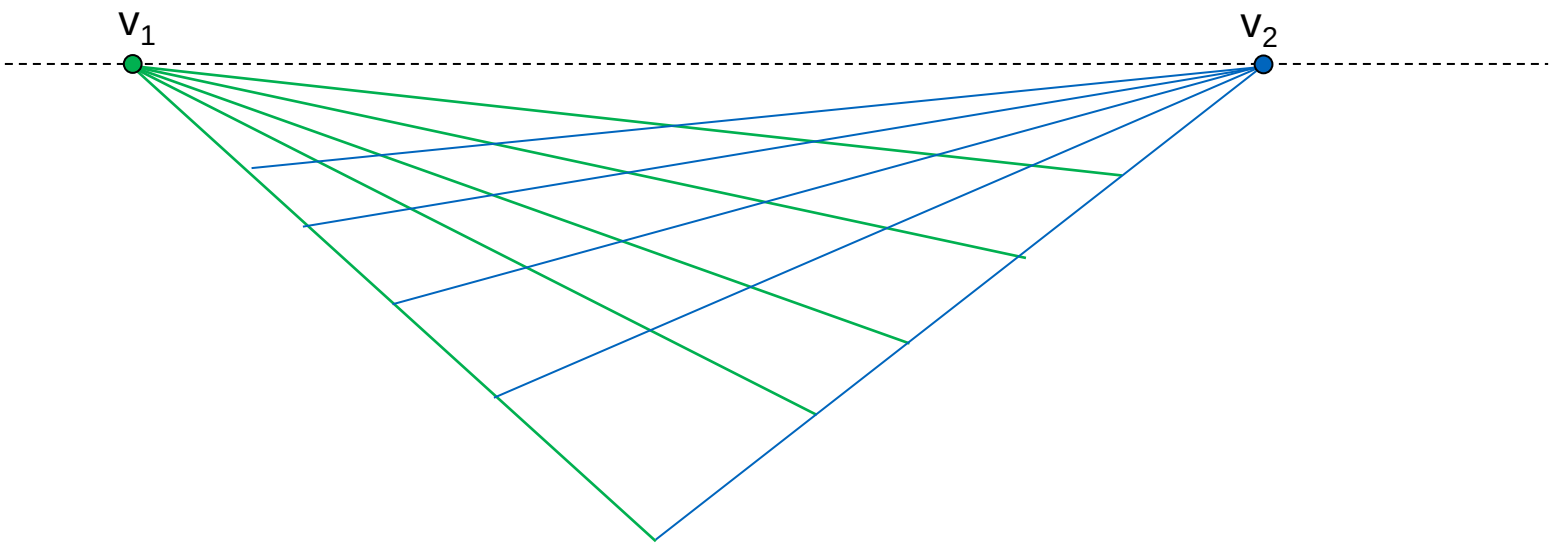
Think of extending the line on the ground plane further and further into infinity.

Vanishing points



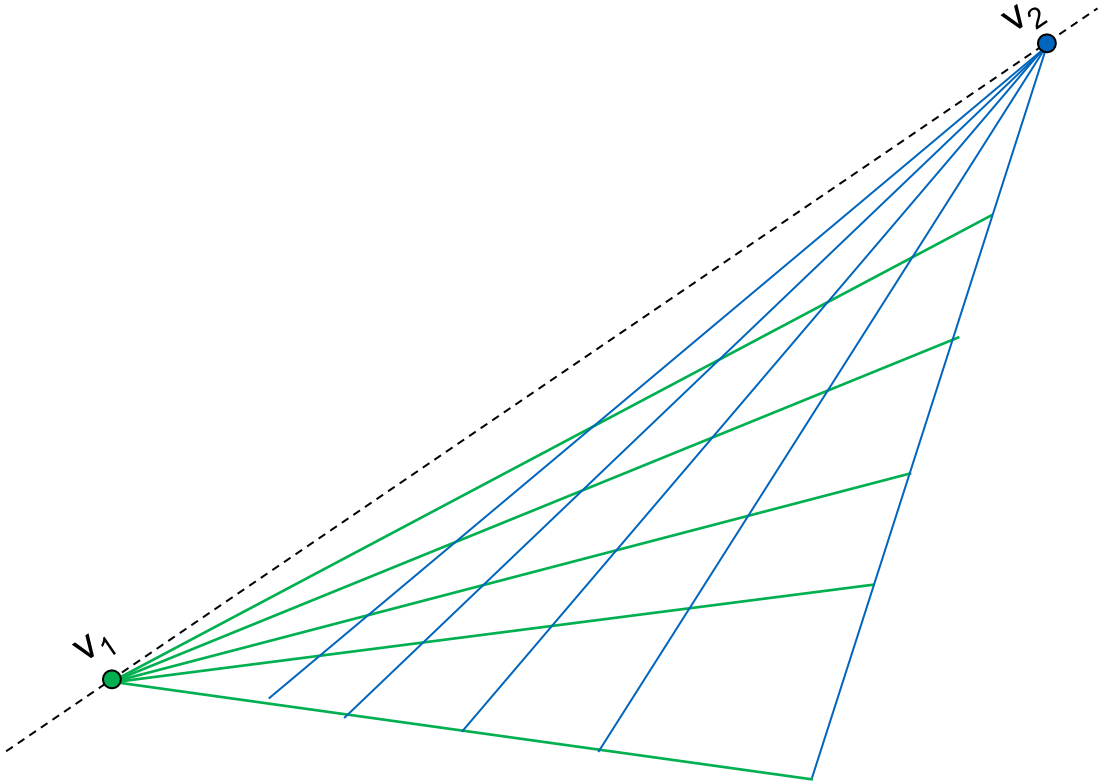
- Any two parallel lines have the same vanishing point **v**
- The vanishing point is the image of the intersection point of the two parallel lines.
- The ray from **C** through **v** is parallel to the lines
- An image may have more than one vanishing point

Vanishing lines



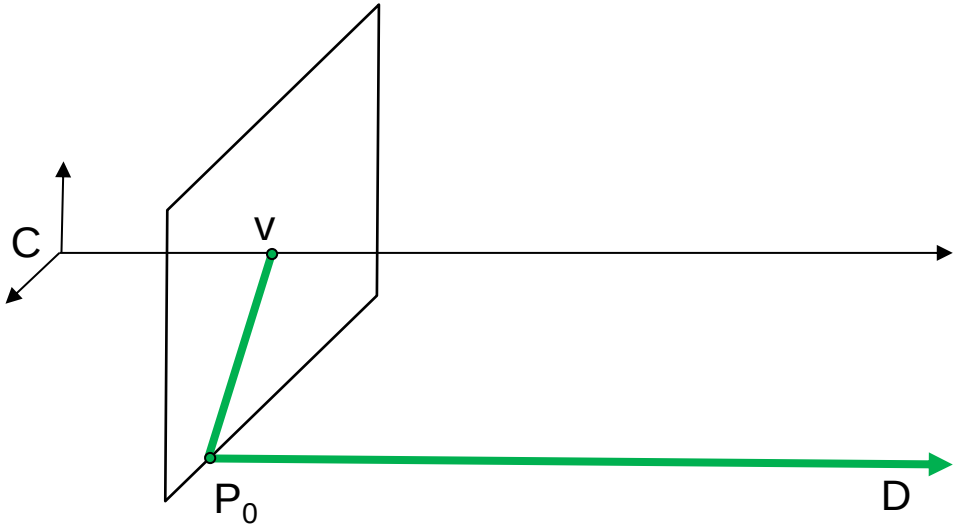
- Multiple vanishing points
  - Any set of parallel lines on the plane define a vanishing point
  - Lines at different orientation result in a different vanishing point
  - The union of all the vanishing points from lines on the same plane is the vanishing line

Vanishing lines



- Different planes define different vanishing lines.

Computing vanishing points

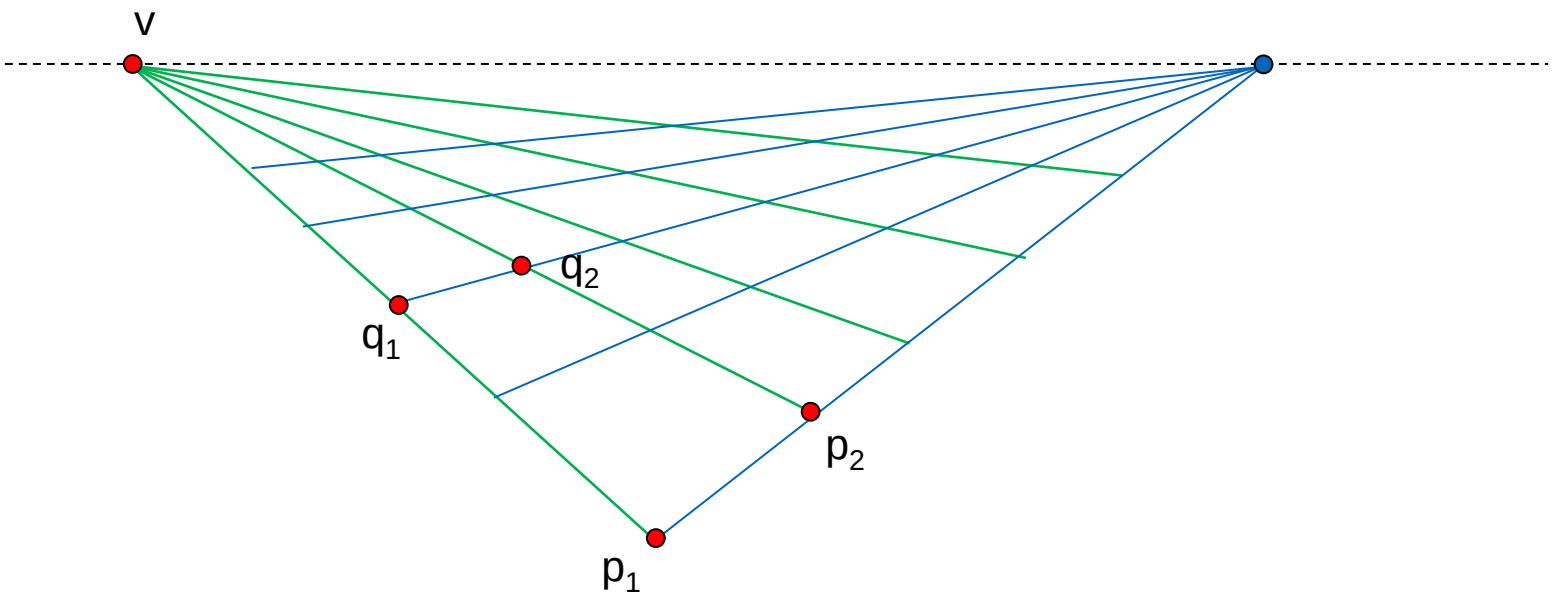


$$\mathbf{P}(t) = \mathbf{P}_0 + t\mathbf{D}$$

$$\mathbf{P}(t) = \begin{bmatrix} P_X + tD_X \\ P_Y + tD_Y \\ P_Z + tD_Z \\ 1 \end{bmatrix} \cong \begin{bmatrix} P_X/t + D_X \\ P_Y/t + D_Y \\ P_Z/t + D_Z \\ 1/t \end{bmatrix} \quad t \to \infty \quad \mathbf{P}_\infty \cong \begin{bmatrix} D_X \\ D_Y \\ D_Z \\ 0 \end{bmatrix}$$

- Properties $\mathbf{v} = \Pi\mathbf{P}_\infty$
  - $\mathbf{P}_\infty$ is a point at infinity, **v** is its projection
  - They depend only on line direction
  - Parallel lines $\mathbf{P}_0 + t\mathbf{D}$, $\mathbf{P}_1 + t\mathbf{D}$ intersect at $\mathbf{P}_\infty$

Computing vanishing points (from lines)



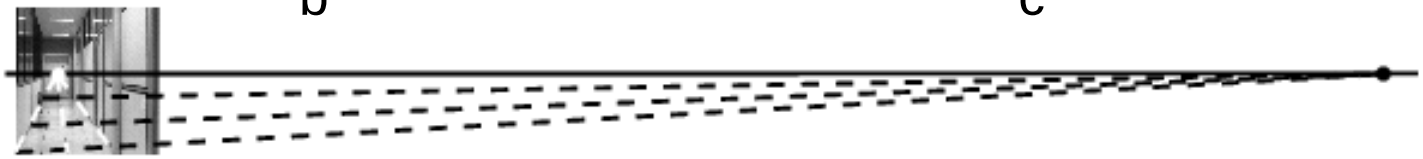
- Intersect p_1q_1 with p_2q_2

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

Real vanishing points

b

c

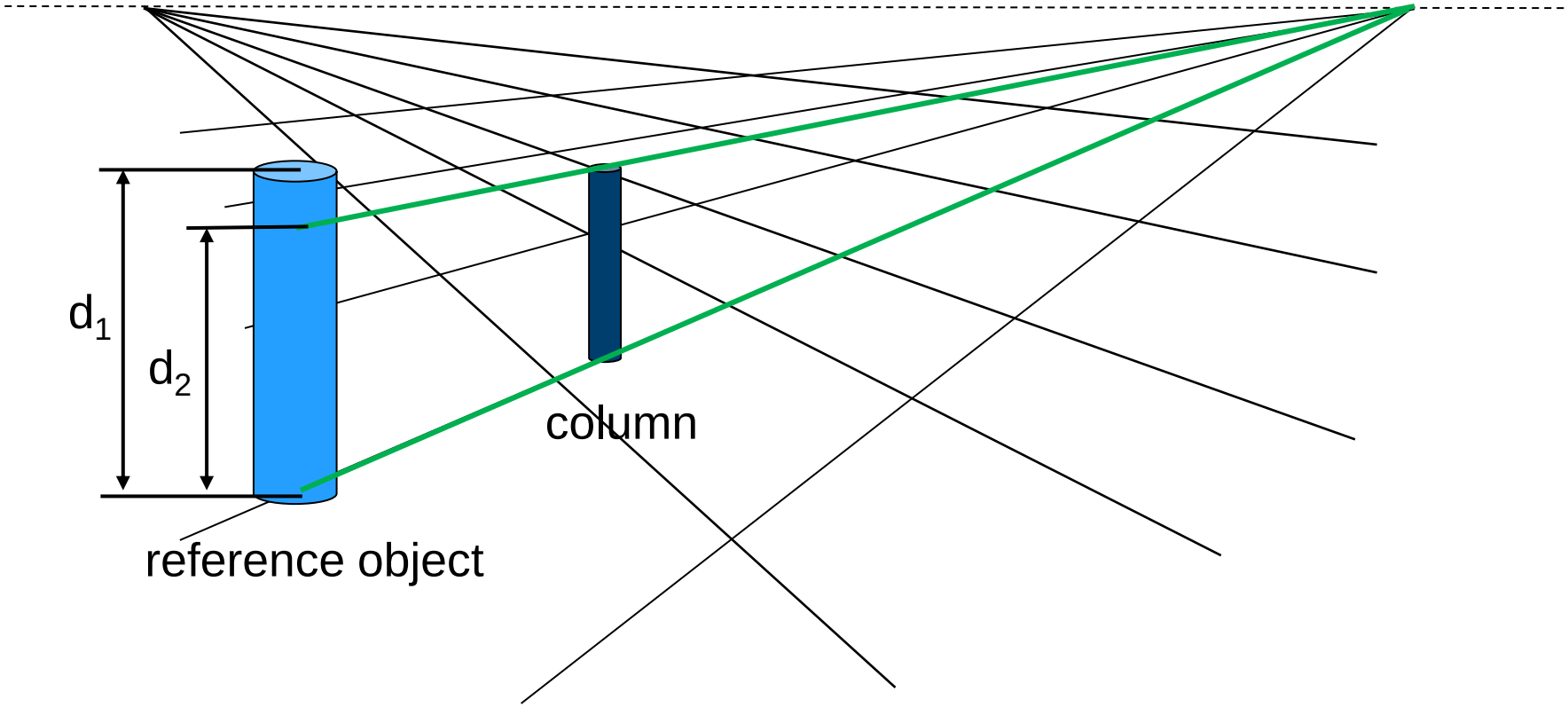


a

[Image source: Richard Hartley and Andrew Zisserman]

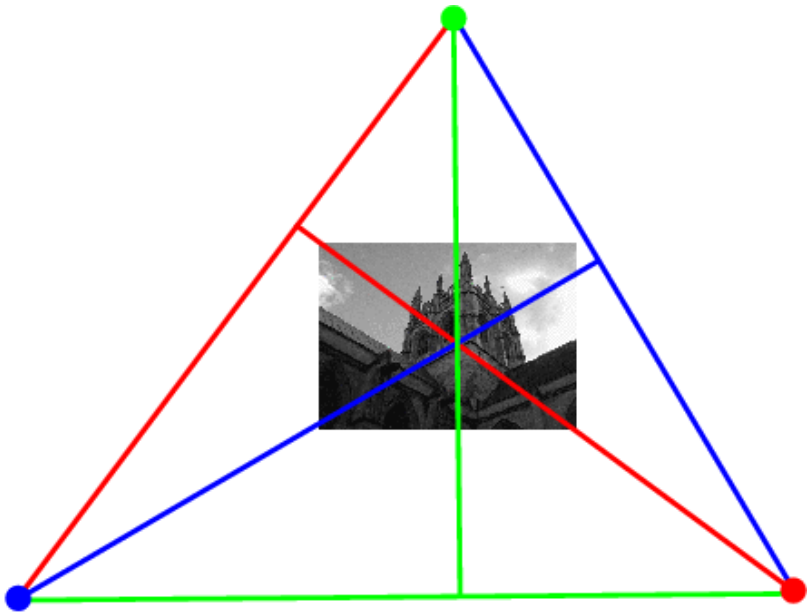
Vanishing point of a line parallel to a plane lies on the vanishing line of the plane

Measuring heights using vanishing points



Height column = height of reference object*d_2/d_1

Camera calibration from orthogonal vanishing points



[Image source: Richard Hartley and Andrew Zisserman]

Recap - Learning goals

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