# Robot Vision:
# Camera calibration

Prof. Friedrich Fraundorfer

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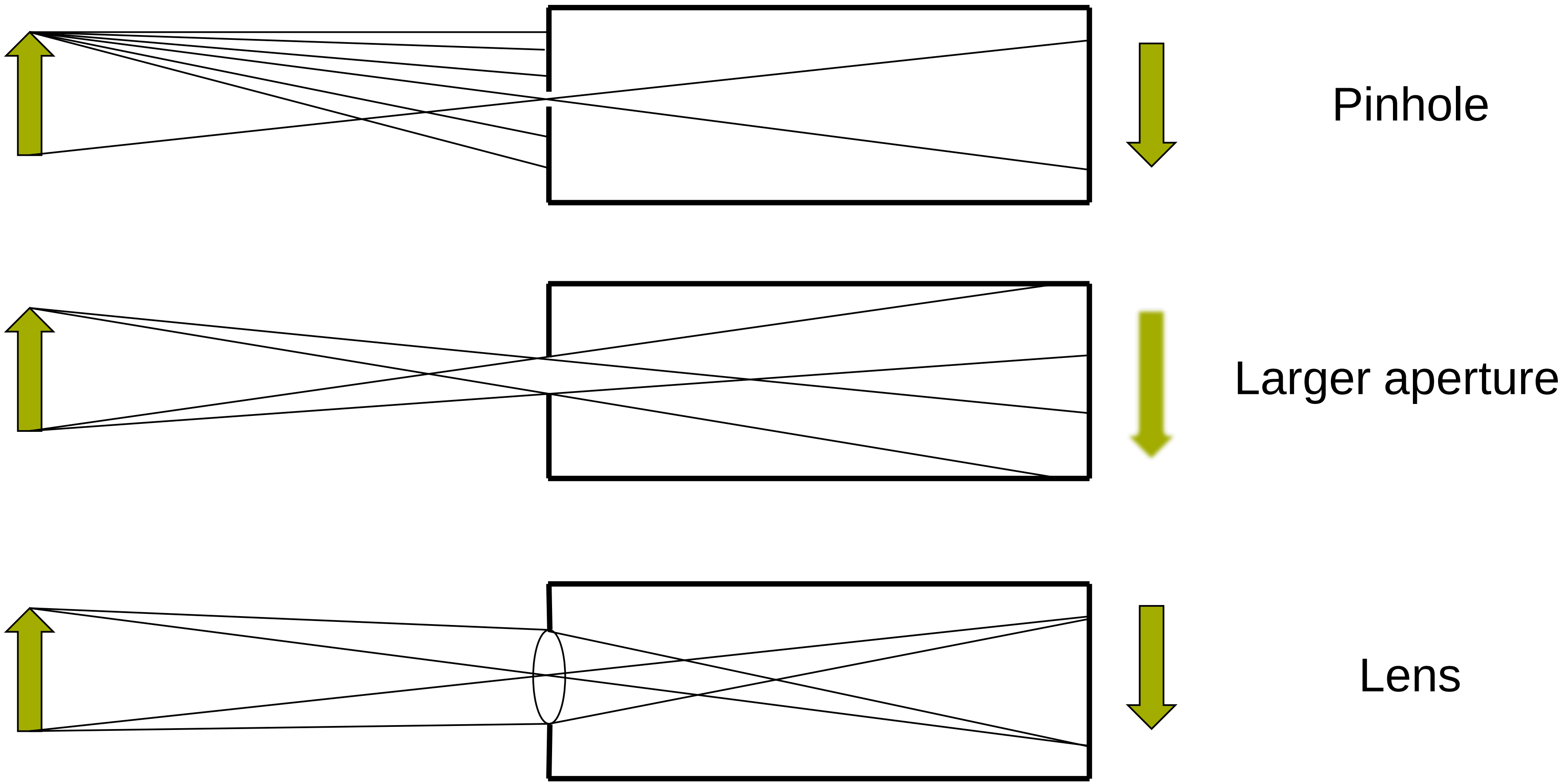
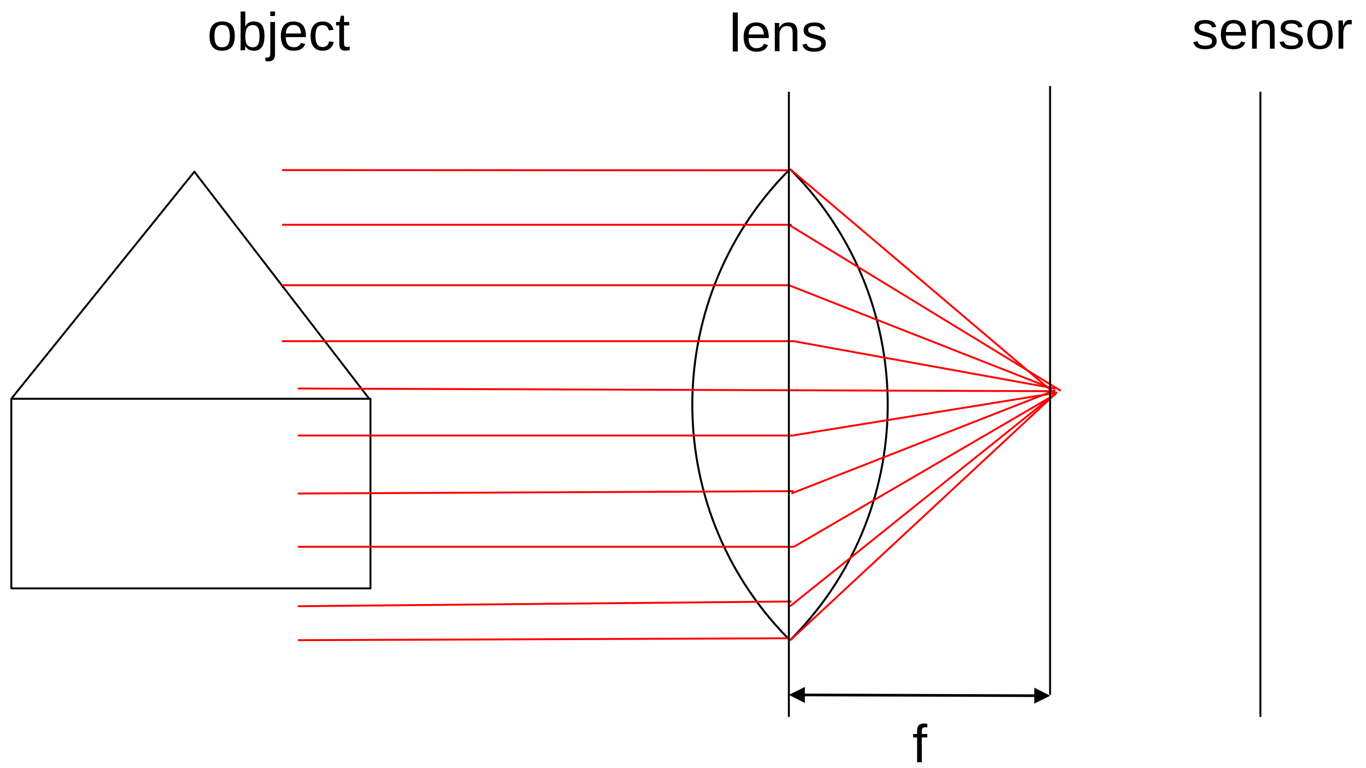
# Outline

- Camera calibration
  - Cameras with lenses
  - Properties of real lenses (distortions, focal length, field-of-view)
  - Calibration algorithm using planar targets (Zhang)

# Pinhole camera and aperture

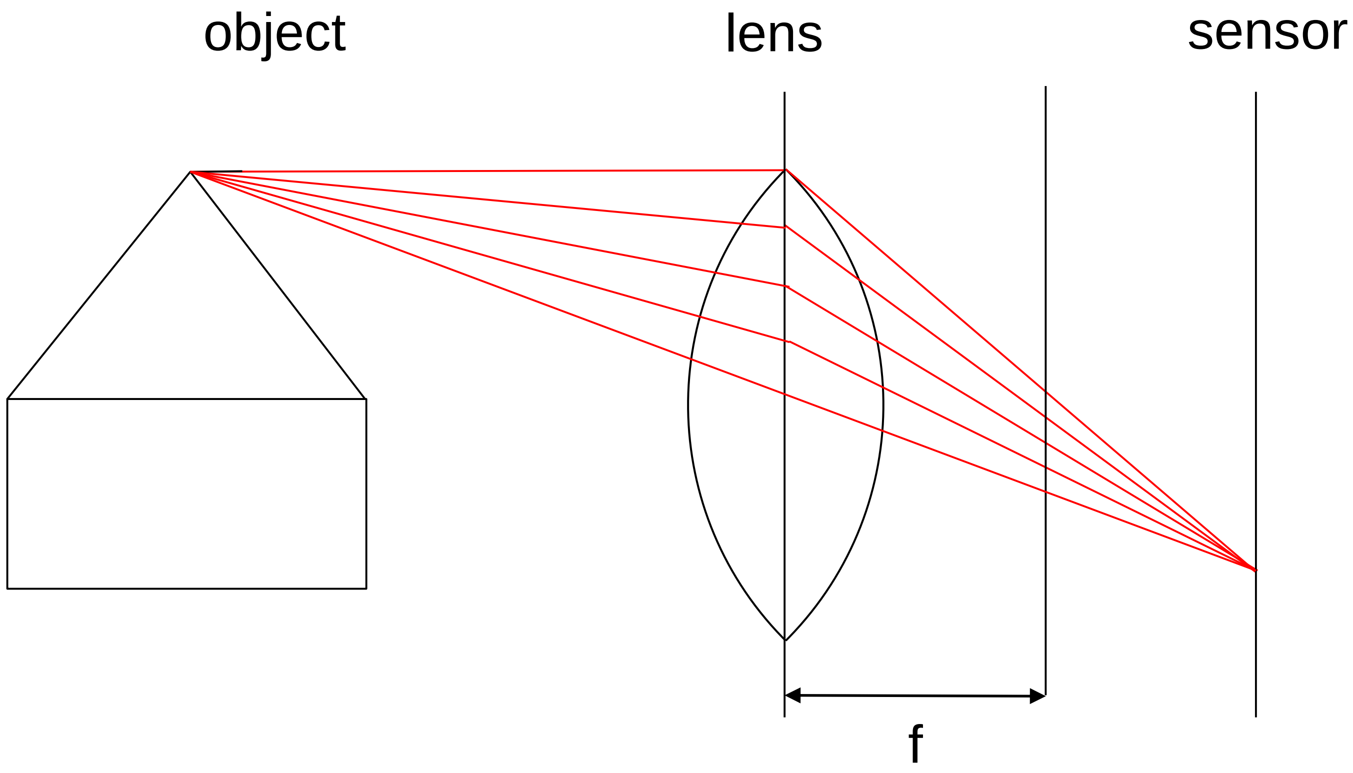


# Effect of a lens



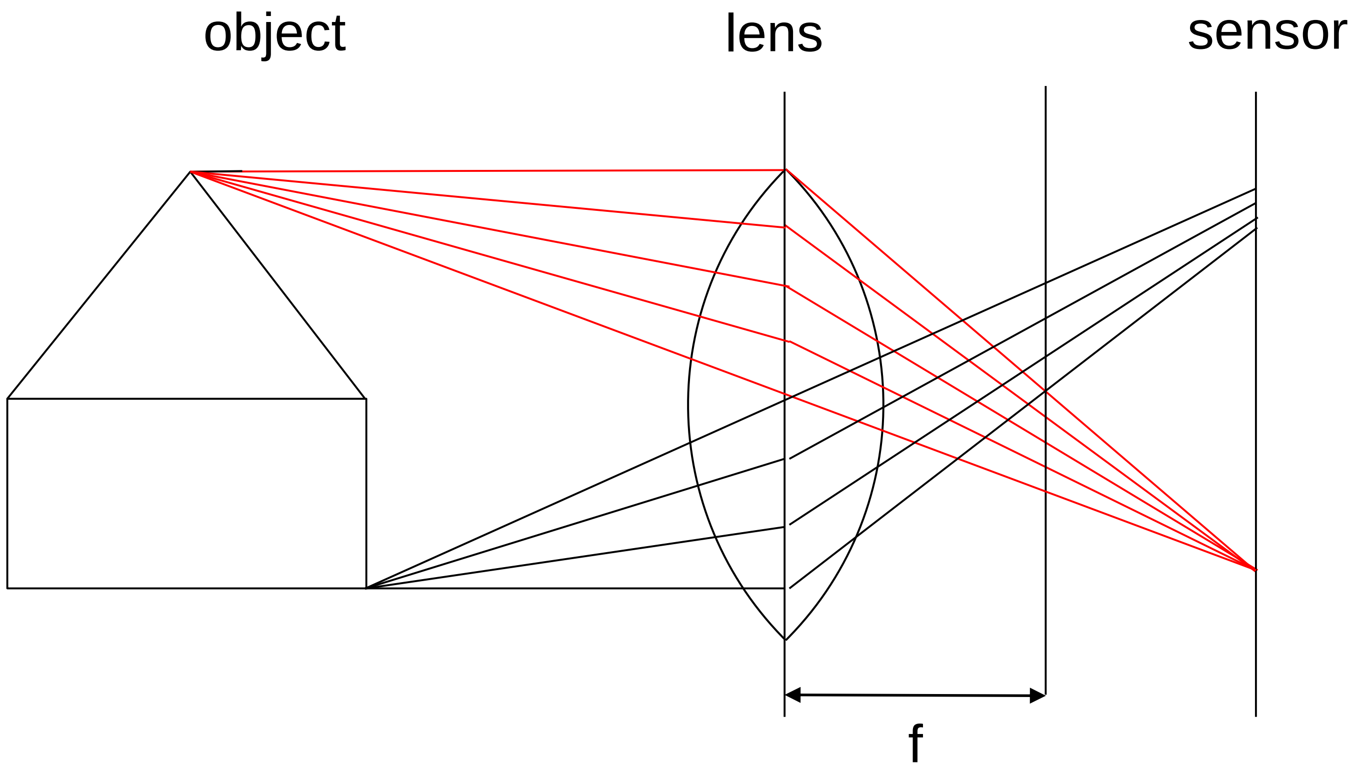

- Thin lens model:
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point at distance f

# Effect of a lens



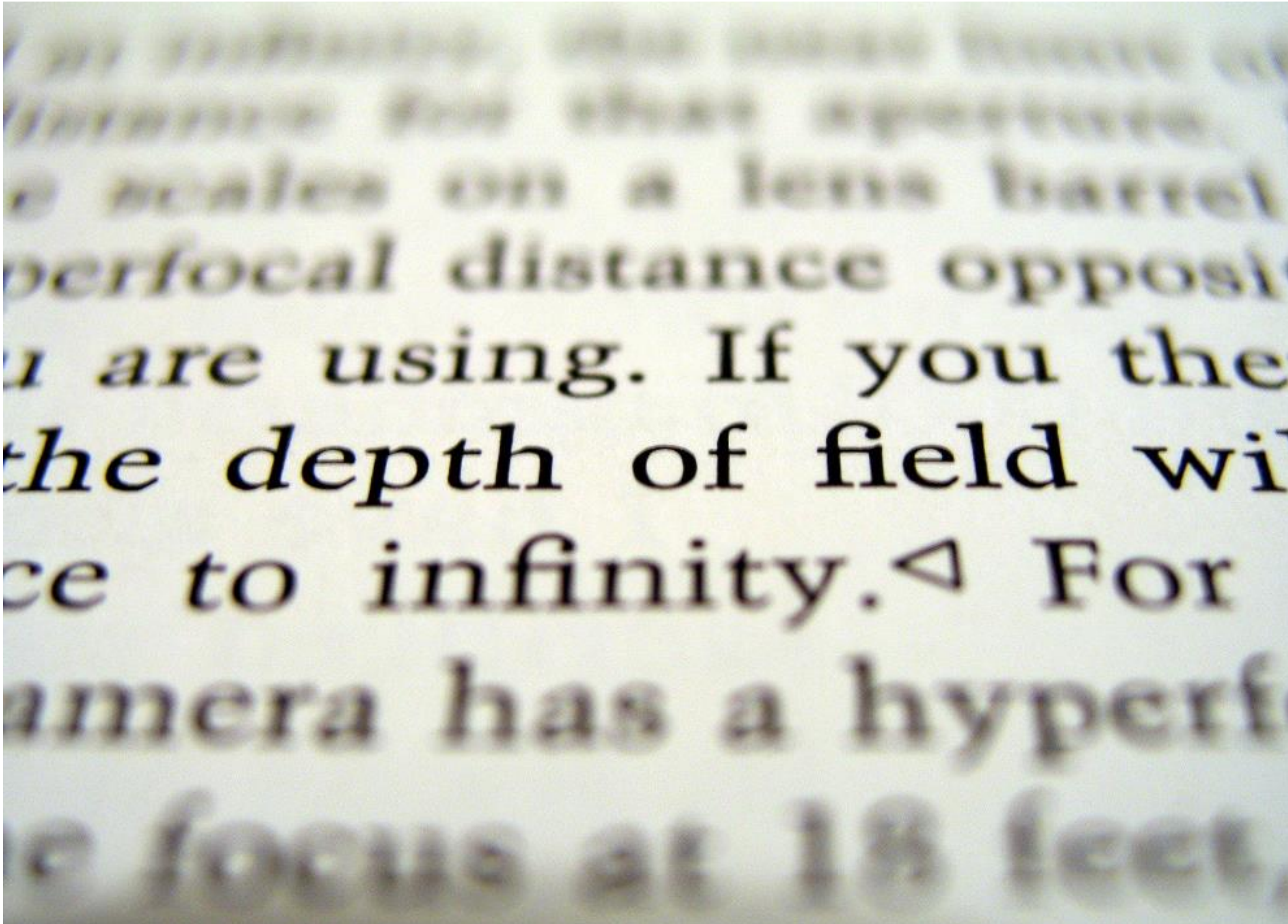

- Thin lens model:
  - Rays passing through the center are not deviated, equivalent to pinhole model

# Effect of a lens



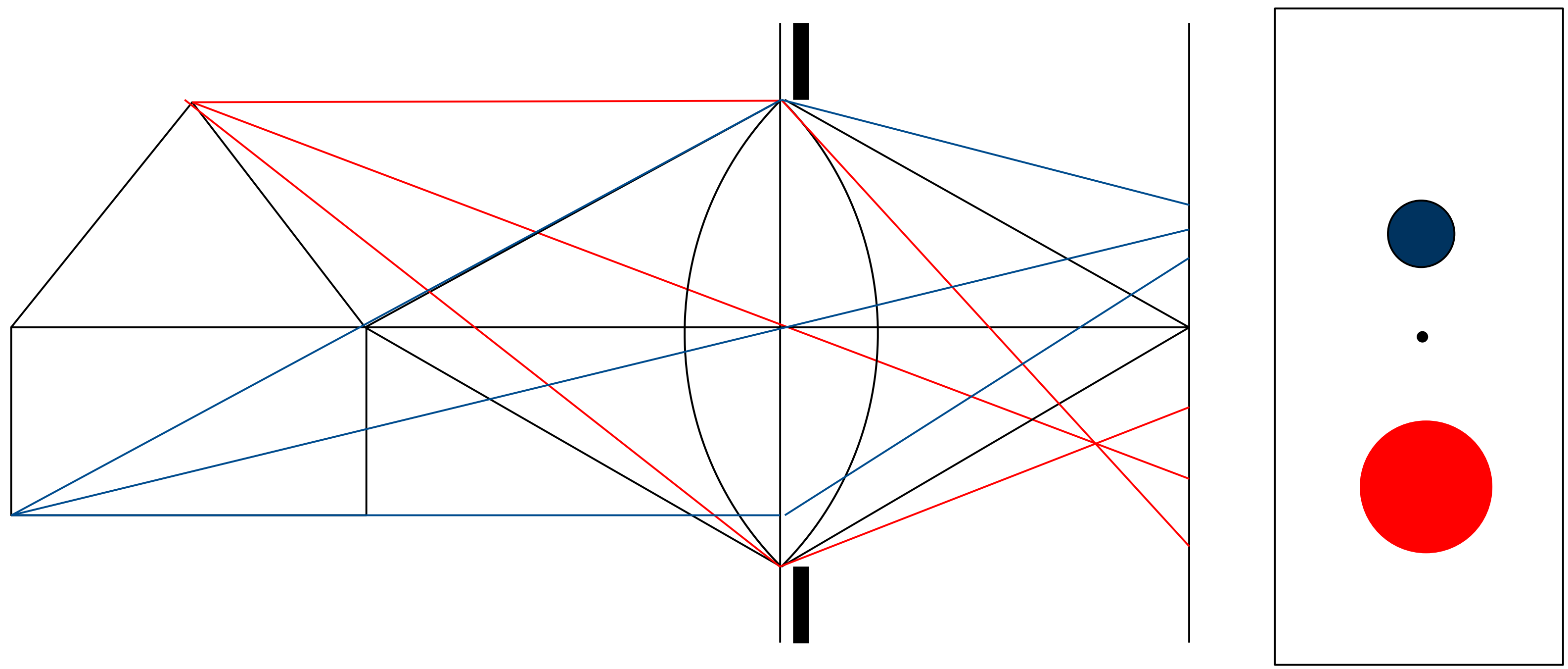

- Thin lens model:
  - Light rays originating from different depths meet at different locations, only images from specific distances are in focus

# Depth of Field/Focal depth

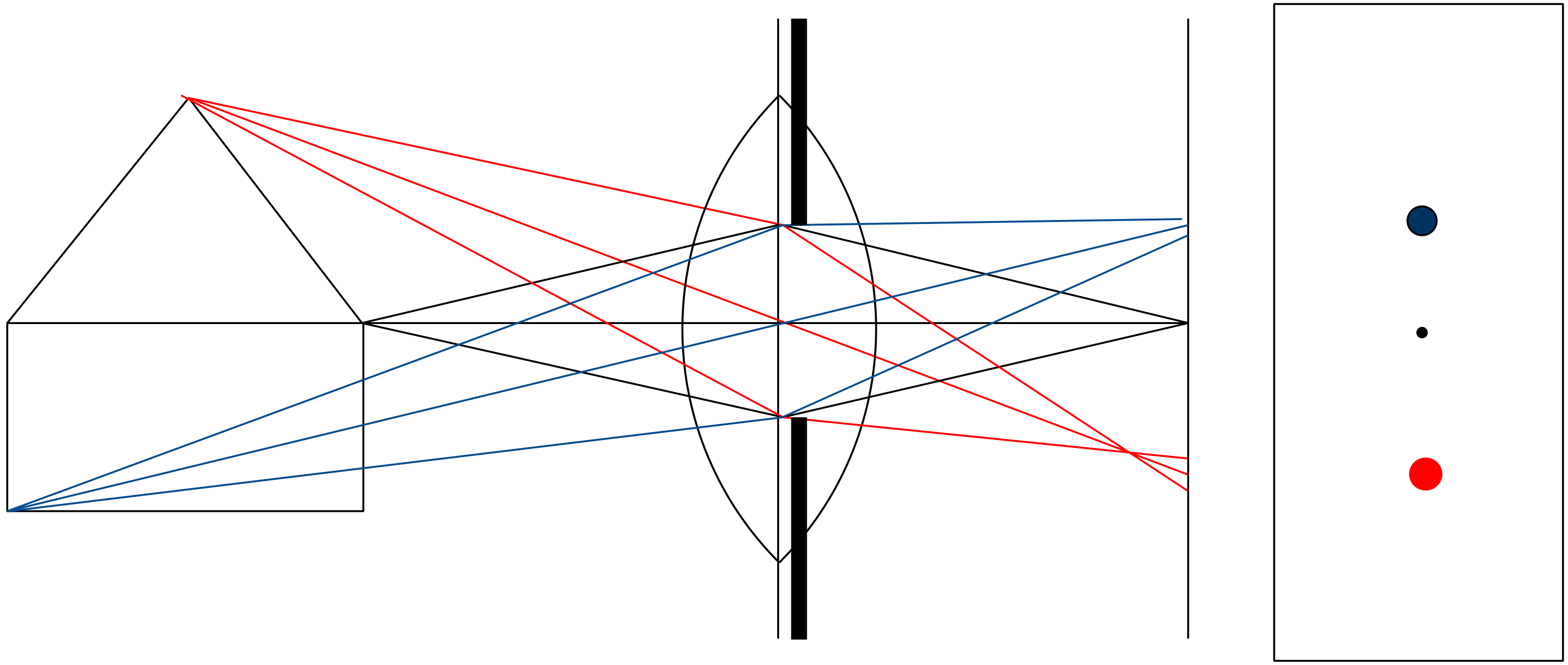



Source: Wikipedia (CC BY-SA 3.0)

# Aperture controls depth of field



# Aperture controls depth of field



- A smaller aperture increases the range in which the object is approximately in focus
- But a small aperture reduces the amount of light that reaches the sensor (larger exposure time needed)

# Field of view/Angle of view


84 deg

63 deg


47 deg

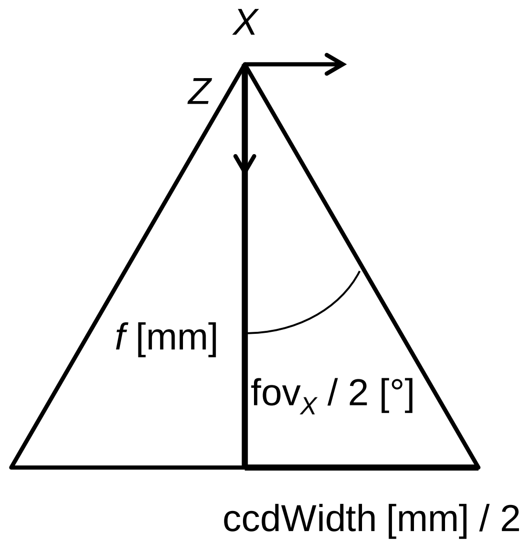
34 deg

Source: Wikipedia (Public domain)

# Field of view (FOV) calculation

- FOV depends on focal length and chip size

$fov_X$ [rad]
focal length $f$ [mm]
ccdWidth [mm]

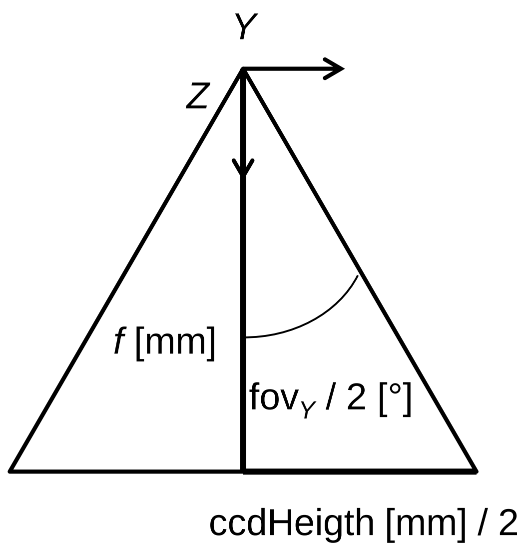



$$fov_X = 2 * \tan^{-1}((ccdWidth / 2) / f)$$

# Field-of-view (FOV)

- CCD chip is not quadratic, FOV is different in x/y direction

$fov_Y$ [rad]
focal length $f$ [mm]
ccdHeight [mm]




$$fov_Y = 2 * \tan^{-1}((ccdHeigth / 2) / f)$$

# Lens distortions - Radial and tangential

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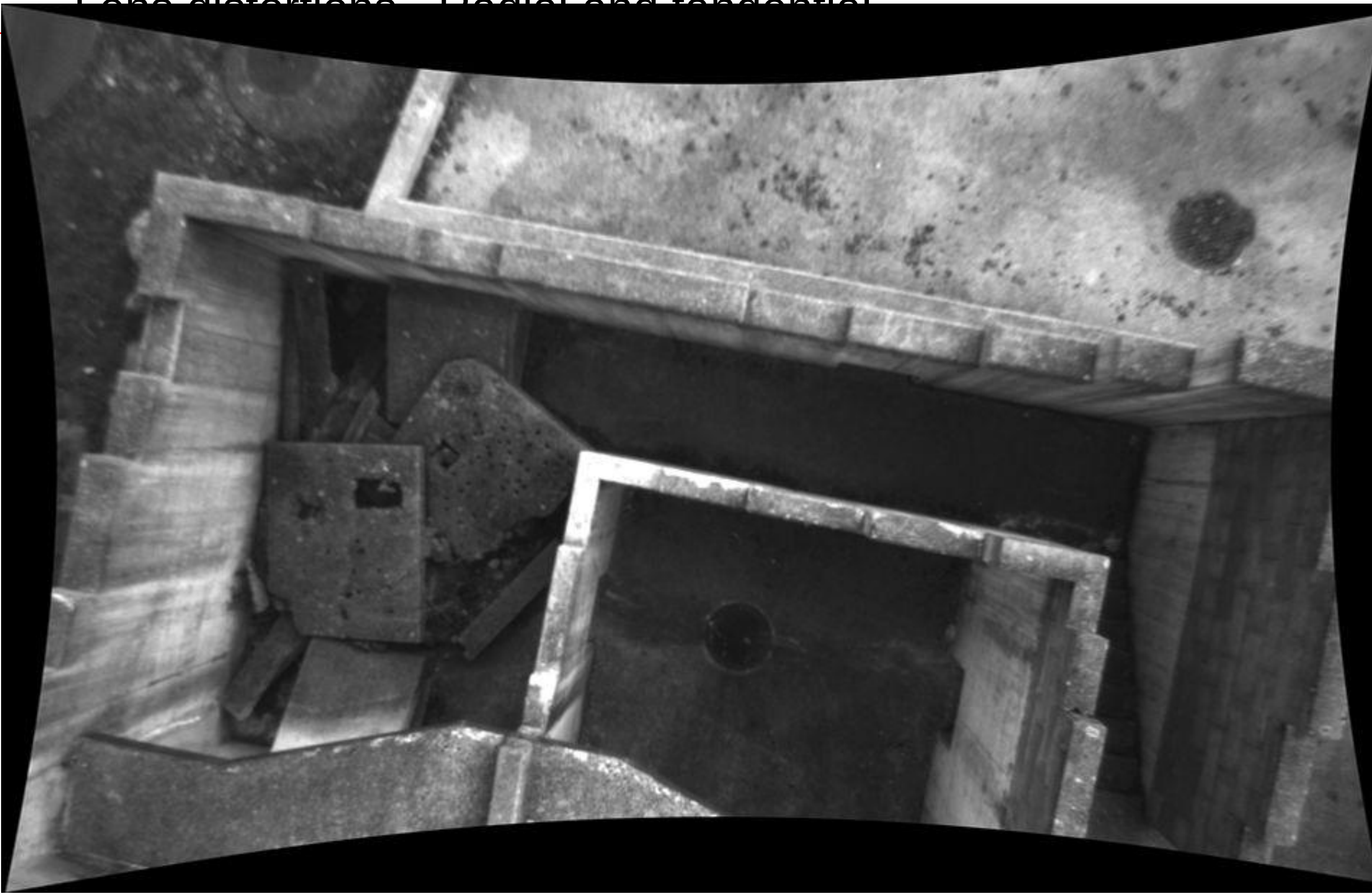


# Strong radial distortion

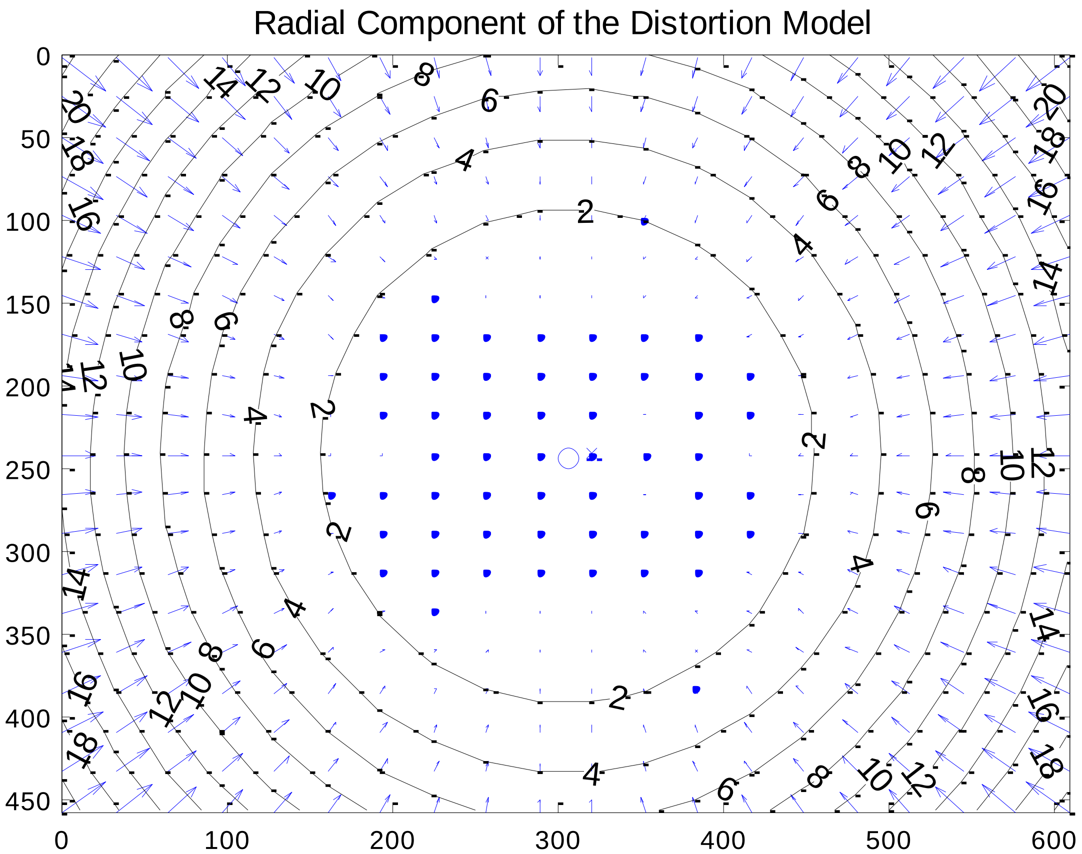
# Mathematical model for radial distortion

$$x_{corrected} = x(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

$$y_{corrected} = y(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

- $x_{corrected}$, $y_{corrected}$ … undistorted normalized image coordinate
- x,y … normalized measured image coordinate (distorted coordinate)
- r .. distance to image center $r^2 = x^2 + y^2$
- k-values .. distortion coefficients

# Example values




| | | |
|---|---|---|
| Pixel error | = [0.5906, 0.4218] | |
| Focal Length | = (662.495, 664.678) | +/- [1.434, 1.543] |
| Principal Point | = (306.513, 241.751) | +/- [2.835, 2.608] |
| Skew | = 0 | +/- 0 |
| Radial coefficients | = (-0.2791, 0.3203, 0) | +/- [0.01144, 0.04729, 0] |
| Tangential coefficients | = (0.0005044, 0.0002783) | +/- [0.0006436, 0.0006694] |

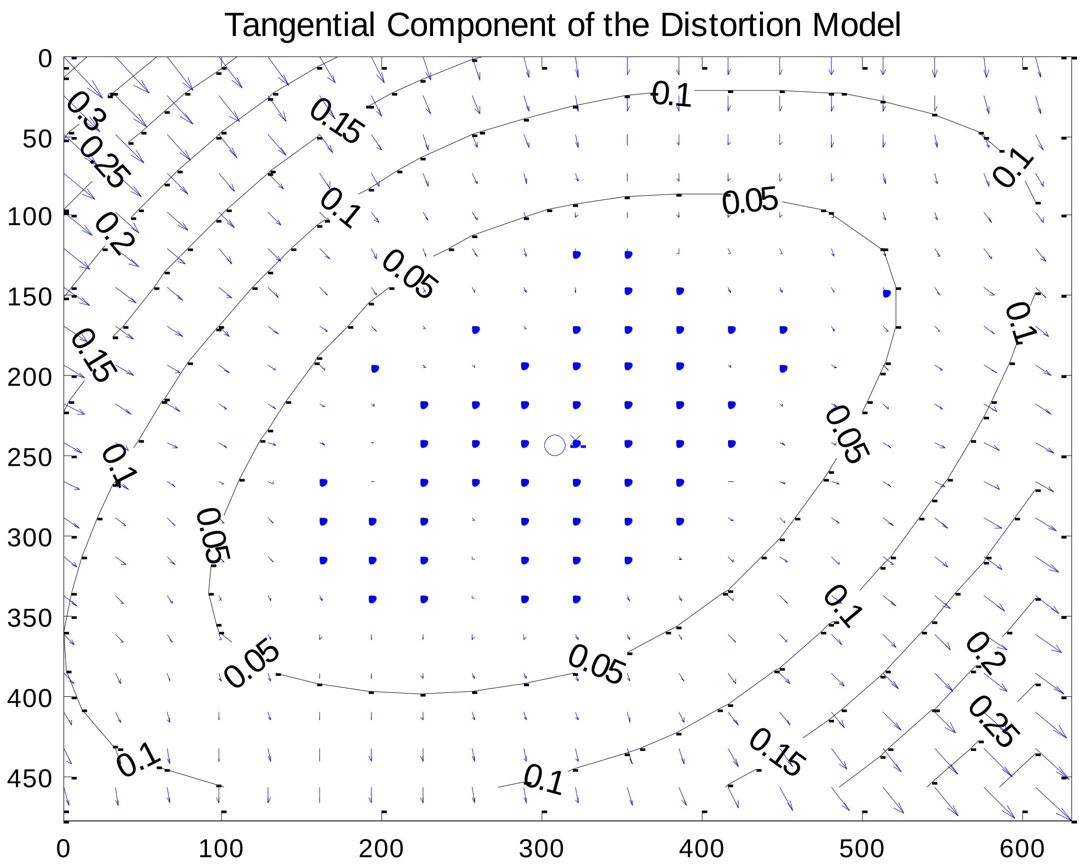
# Mathematical model for tangential distortion

$$x_{corrected} = x + [2p_1 y + p_2(r^2 + 2x^2)]$$

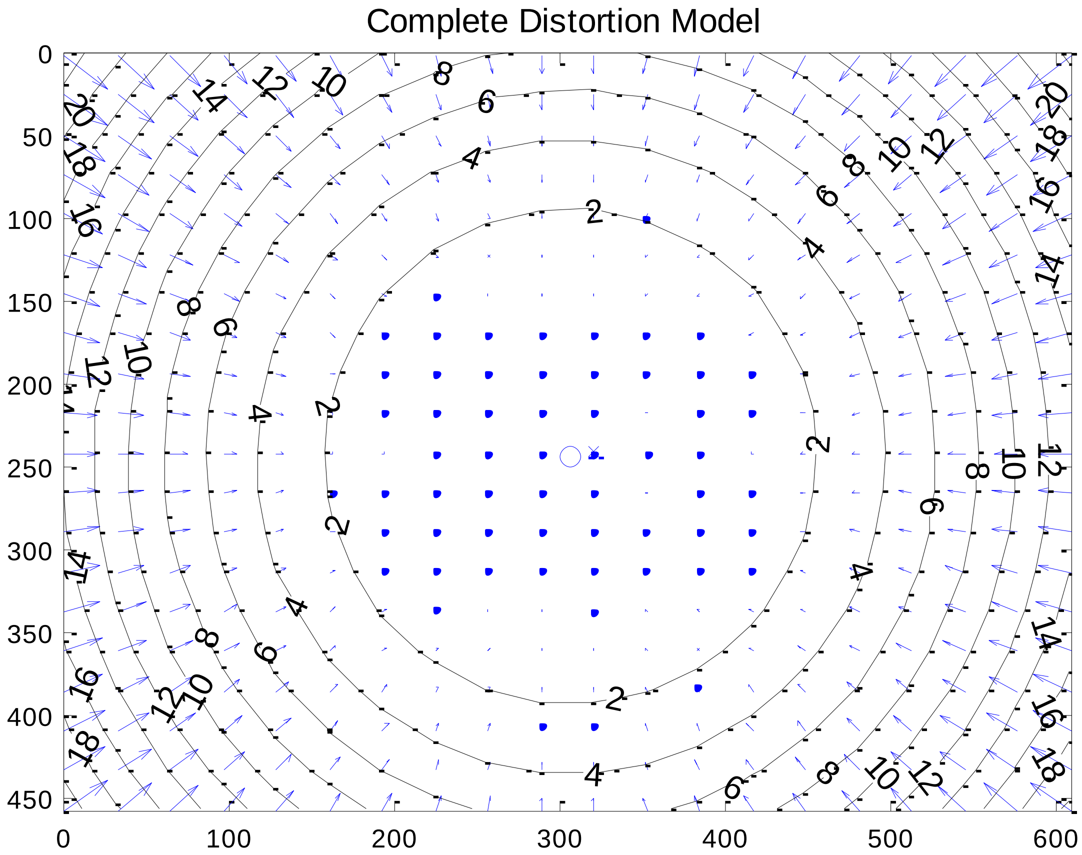
$$y_{corrected} = y + [p_1(r^2 + 2y^2) + 2p_2 x]$$

- $x_{corrected}$, $y_{corrected}$ … undistorted normalized image coordinate
- x,y … normalized measured image coordinate (distorted coordinate)
- r .. distance to image center $r^2 = x^2 + y^2$
- p-values .. distortion coefficients

# Example values



# Example values

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# Undistort images

$$\begin{pmatrix} x_{corrected} \\ y_{corrected} \\ 1 \end{pmatrix} = K\, rect\left( K^{-1} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \right)$$

- $x_{corrected}$, $y_{corrected}$ … undistorted normalized image coordinate
- x,y … measured image coordinate (distorted coordinate)

- To render an undistorted image it is best to use target-to-source warping, but this needs the inverse of the undistortion function (does not exist in closed form)
- Many geometric algorithms just undistort a few feature points

# The calibration procedure

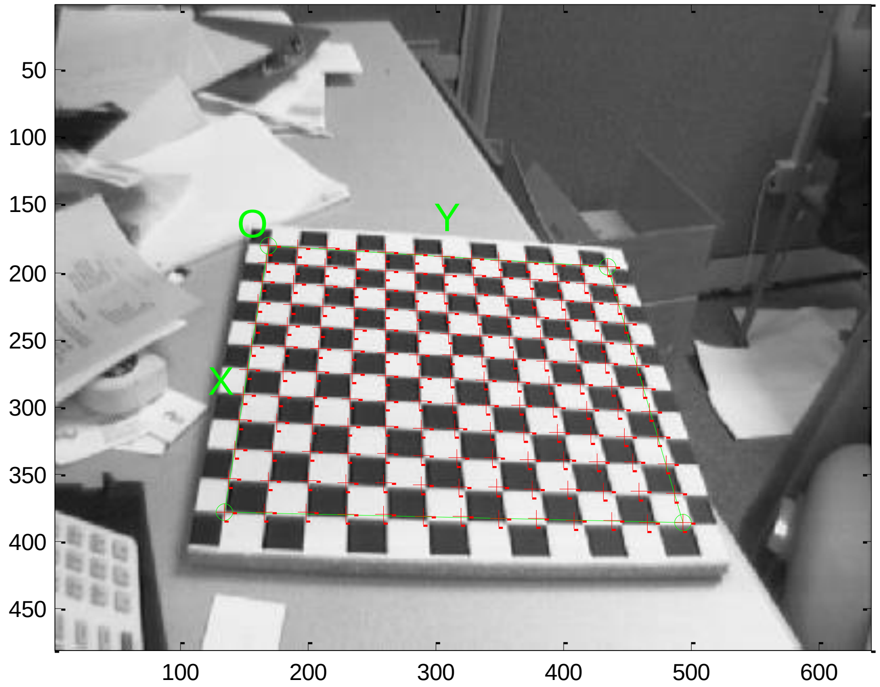
- Calibration needs to estimate intrinsics and distortion parameters
  - Intrinsics ($f_x, f_y, c_x, c_y$)
  - Distortion parameters ($k_1, k_2, k_3, p_1, p_2$)

- Method:
  1. Estimate intrinsics first assuming that there are no distortions (does not work for images with strong distortions)
  2. Estimate distortion parameters with fixed values for intrinsic
  3. Refine all estimates at the same time using non-linear optimization

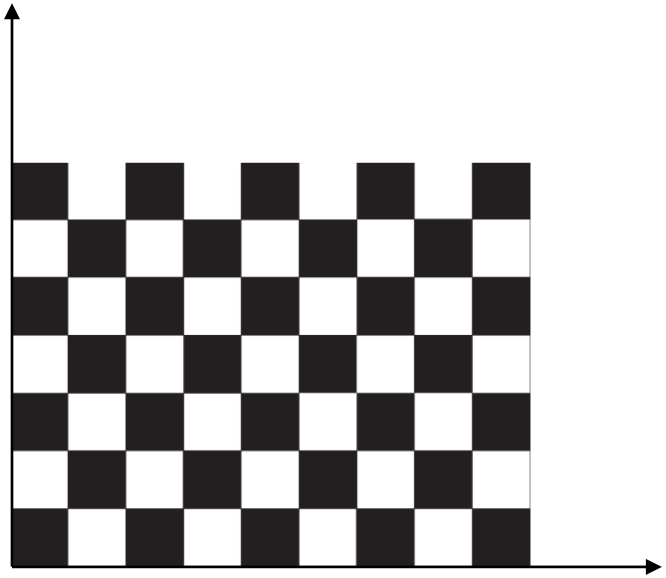
# Calibration from planar target

- Method from Zhang 1999, Flexible Calibration by Viewing a Plane From Unknown Orientations

image




object with known dimensions



$$x = K[R\ t]X$$

$$\min_{K,R,t} \sum_{i=1}^{n} \sum_{j=1}^{m} \left\| x_{ij} - P_j X_i \right\|^2$$

# Calibration from planar target: Algorithm

1. Find all matches between x and X in all images
2. Compute homographies between x and X (represent camera poses)
3. Compute initial values for intrinsics from these homographies (see Zhang) by solving a linear equation system ignoring distortion
4. Estimate distortion using non-linear optimization using fixed intrinsics (can start optimization using 0 values for distortion)

$$\min_{p} \sum_{i=1}^{n} \sum_{j=1}^{m} \left\| x_{ij} - dist(P_j X_i; p) \right\|^2$$

5. Re-estimate all parameters (distortion and instrinsics) using non-linear optimization

$$\min_{K,R,t,p} \sum_{i=1}^{n} \sum_{j=1}^{m} \left\| x_{ij} - dist(P_j X_i; p) \right\|^2$$